

## Overview

- Asymptotic notation review
- Runtime analysis example
- Two approaches to  $n \in \omega\left(2^{\sqrt{\lg n}}\right)$
- Little- $o$  characterization
- Memory model review
- Bounds with integration

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## Problems

**Q1.** Show that  $n \in \omega\left(2^{\sqrt{\lg n}}\right)$  using the definition.

**Q2.** Give a tight bound on the runtime of the following algorithm as a function of  $n$ .

```
k = 1
for(i = 1; i <= n; ++i):
    j = 0
    while j <= n:
        j += k
    k *= 2
```

**Q3.** Let  $f(n), g(n)$  be eventually positive. In lecture we saw that if  $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$  then  $f(n) \in o(g(n))$ . Prove the converse.

**Q4.** Give an exact bound on the  $n$ -th Harmonic number  $H_n := \sum_{k=1}^n 1/k$ .

Note: this question requires bounds with integration.

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## Additional problems

**Q5.** Give asymptotic upper and lower bounds (make them as tight as you can) for  $T(n)$  if,

(a)  $T(n) = 3T(n/3) + n/\lg n$

(b)  $T(n) = T(n-1) + 1/n$

(c)  $T(n) = \sqrt{n}T(\sqrt{n}) + n$