Math session

All sample solutions can be found in `mathsessionsolutions.pdf`.

Question 1: You have a list of n integers that you wish to split into equal-size pieces, or rather, as-close-as-possible-to-equal-size pieces, as the size of the list may not be divisible by 3. Express the lengths of the biggest and the smallest pieces as integers.

Question 2: Suppose you wish to form a list of k items from a set of n distinct items, where each item can appear at most once in the list. Two lists are different if they either do not contain the same items or if they have the same items in a different order. How many different lists can you form?

Question 3: Show that $\sum_{i=2}^{n} \log i \in \Theta(n \log n)$ *without* determining an exact value for the sum.

Question 4: Express $\log((mn)!)$ in order notation as a function of m and n.

Question 5: In a *d-ary* tree, each internal node can have at most d children.

1. What is the maximum number of leaves in a d-ary tree of height h?

2. What is the minimum number of leaves in a d-ary tree of height h?

3. What is the maximum number of nodes in a d-ary tree of height h?

4. What is the minimum number of nodes in a d-ary tree of height h?

5. What is the maximum height of a d-ary tree with ℓ leaves?

6. What is the minimum height of a d-ary tree with ℓ leaves?

Question 6: In all of the following subquestions, $P(x)$ is the predicate "The integer x is odd" and $Q(x)$ is the predicate "The integer x is greater than 10."

1. Prove that the following statement is false: "There exists an x such that $P(x)$ is true and $P(x + 1)$ is true."

2. Prove that the following statement is true: "There exists an x such that $Q(x)$ is true and $Q(x - 1)$ is true."

3. What kind of statement is the following? "For every number x greater than 5, $Q(x)$ is true." Prove or disprove it.

Question 7: Suppose you wished to use induction to prove that a tree with height n has at most 2^n leaves.

1. What is the base case?

2. Provide a proof of the base case.

3. What is the induction step?

4. Provide a proof of the induction step.

Question 8: Determine the expected value of the weight of an edge in Sample graph 4, where an event is the selection of an edge, each edge is equally likely, and the value of an event is the weight of the selected edge.