CS 234: Data Types and Structures

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Module 5

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Case study

Problem: When colour is applied to a part of a web page, what other parts of the page will obtain the same colour?

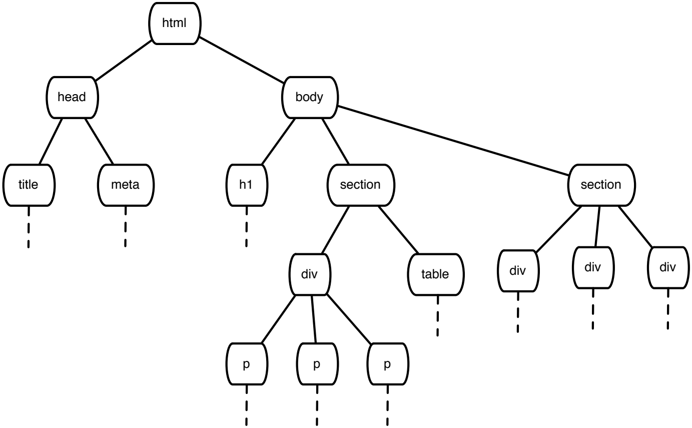
For example, if a section is coloured, the paragraphs and lists will get the same colour.

Recipe for user/plan

1. Determine types of data and operations.
2. For each type, choose/modify/create an ADT.
3. Develop pseudocode algorithm using ADT operations.
4. Calculate cost of algorithm with respect to costs of operations.
5. Using information from provider, choose best option.

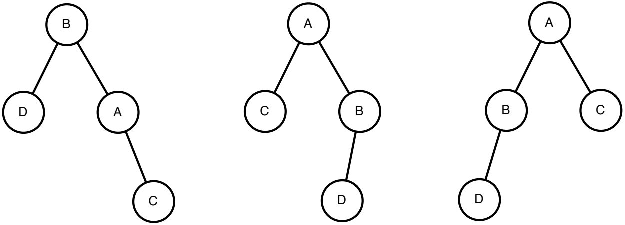
Representing a web page

The Document Object Model represents a web page as a tree.



Tree review

Which of these trees are the same?



Basic definitions

A **tree** is formed of **nodes** connected by **edges**. (This is not the same as a node in a linked list.)

In a **rooted tree**, one node is designated as the **root** of the tree.

In a drawing where the root is at the top, an edge connects a **parent** to a **child**, where the parent is the node closer to the root.

Nodes that share a parent are **siblings**.

A node without children is a **leaf**; a node that is not a leaf is an **internal node**.

A node's parent, its parent's parent, and so on up to the root are its **ancestors**; a node's children, children's children, and so on are its **descendants**.

A node and all its descendants form the **subtree rooted at** that node.

Types of rooted trees

A tree is **unordered** if there is no order specified on the children of a node, and **ordered** otherwise.

A **binary tree** is a tree in which each parent has at most two children and each child is specified as either a **left child** or a **right child**.
In a binary tree, the subtree rooted at the left child is the **left subtree** and the subtree rooted at the right child is the **right subtree**.

Terminology for rooted trees

The **path** between nodes n_0 and n_k is the sequence of nodes $\{n_0, n_1, \ldots, n_k\}$ such that there is an edge between n_i and n_{i+1} for all $0 \leq i < k$; a path is **simple** if each node appears at most once in the sequence. The **length** of a path is the number of edges in the path.

The **depth of a node** n is the length of the path between n and the root; a root is thus at depth 0. All nodes of the same depth are on the same **level**.

The **height of a node** n is the maximum length of any path between n and a leaf in the subtree rooted at n; a leaf thus has height 0. The **height of a tree** is the height of the root of the tree.

Nodes as positions to store data

In earlier ADTs, we accessed data by position, such as:

- top (ADT Stack)
- front (ADT Queue)
- index (ADT Indexed Sequence)
- rank (ADT Ranking)
- row and column (ADT Grid)

We can navigate in a tree by starting at the root, choosing a child, and so on to find a specific node.

How do we refer to a particular node?

Instead of using the path from the root, we'll associate a unique **ID** with each node. The type of data used for the ID may depend on the data structure implementing the ADT (e.g. index in an array or pointer to a node in a linked structure).

Data stored in nodes

Depending on the application, a node of a tree can store various types of data, such as:

- a value
- a weight
- a colour

For now, we will define our ADTs such that each node stores a single value.

Search operations for trees

- Find the value of a node
- Find the root of the tree
- Find the parent of a node
- Find a specific child of a node
- Find all children of a node
- Find the node storing a particular value
- Find all nodes storing a particular value
- Find all nodes in the tree

Issues to consider:

How can multiple nodes be returned?

Use a Group B ADT that allows us to extract all the data items, possibly in some specific order.

Modification operations for trees

- Add a new node
- Delete a node
- Delete a subtree
- Change the value stored in a node
- Swap values stored in two nodes
- Swap subtrees

Issues to consider:

What remains after a node is deleted?

Initially just delete leaves.

Initially start with binary trees.

ADT Binary Tree, without modifications

Preconditions: For all *B* is a binary tree and *Node* is a node in *B*; for *Root B* is not empty.

Name	Returns
Create()	a new empty binary tree
Is_Empty(B)	*True* if empty, else *False*
Root(B)	root of *B*
Value(B, Node)	value stored in *Node*
Parent(B, Node)	parent of *Node* if any, else *False*
Left_Child(B, Node)	left child of *Node* if any, else *False*
Right_Child(B, Node)	right child of *Node* if any, else *False*

ADT Binary Tree, modifications

Preconditions: For all *B* is a binary tree, *Node* is a node in *B*, and *Data* is a data item; for *ADD_LEAF* either *Par* and *Side* are both empty or *Par* is a node in *B* and *Side* is *Left* or *Right*; for *DELETE_LEAF* *Node* is a leaf.

Postconditions: Mutation by *SET_VALUE* (sets value of *Node* to *Data*), *ADD_LEAF* (creates a new node containing *Data* to replace/add the root if *Par* is empty and to replace/add the *Side* subtree of *Par* otherwise), and *DELETE_LEAF* (deletes *Node*).

Name	Returns
SET_VALUE(B, Node, Data)	
ADD_LEAF(B, Par, Side, Data)	new added node storing *Data*
DELETE_LEAF(B, Node)	

Example of use of ADT Binary Tree operations

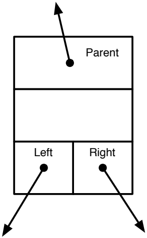
Fruit $\leftarrow$ *CREATE()*
Apple $\leftarrow$ *ADD_LEAF(Fruit, None, None, apple)*
Guava $\leftarrow$ *ADD_LEAF(Fruit, Apple, Left, guava)*
Peach $\leftarrow$ *ADD_LEAF(Fruit, Apple, Right, peach)*
Mango $\leftarrow$ *ADD_LEAF(Fruit, Guava, Right, mango)*
One $\leftarrow$ *ROOT(Fruit)*
Two $\leftarrow$ *PARENT(Fruit, Mango)*
Three $\leftarrow$ *LEFT_CHILD(Fruit, Guava)*
Four $\leftarrow$ *RIGHT_CHILD(Fruit, Guava)*
DELETE_LEAF(Fruit, Peach)
Five $\leftarrow$ *RIGHT_CHILD(Fruit, Apple)*

Linked implementation of ADT Binary Tree

Data structures:

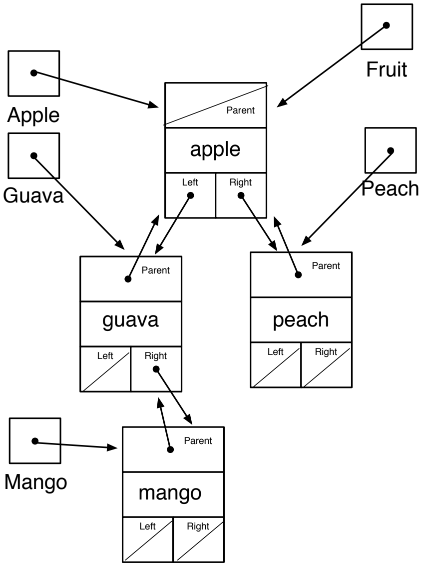
- Variable pointing to root node, if any
- Nodes storing data items and three pointers *Parent* (to parent), *Left* (to left child), and *Right* (to right child)

Worst-case running times of operations are all in $\Theta(1)$.
Cost of searching for a node from the root depends on depth.

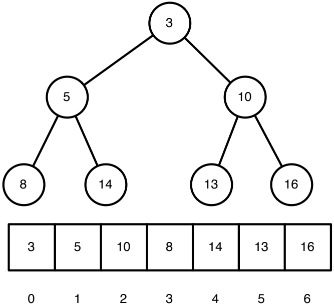


Caution: The word “node” can mean either or both of “node in a tree” and “node in a linked implementation.”

Example illustrated



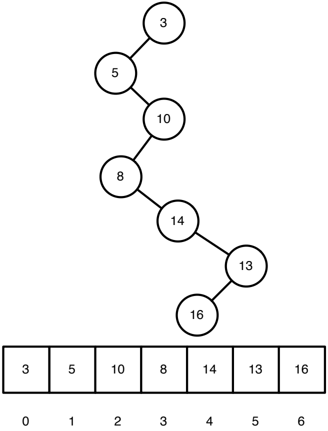
Contiguous implementation of ADT Binary Tree



Observations:

- For node at index p, index of left child is $2p+1$
- For node at index p, index of right child is $2p+2$
- For node at index p, index of parent is $\lfloor (p-1)/2 \rfloor$

Exploring a contiguous implementation



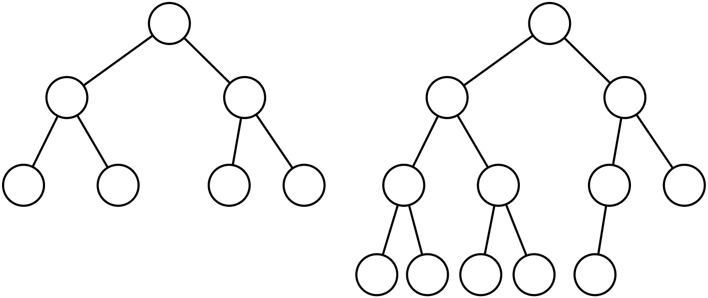
Observations:

- For node at index p, index of left child may not be $2p+1$
- For node at index p, index of right child may not be $2p+2$
- For node at index p, index of parent may not be $\lfloor (p-1)/2 \rfloor$

More terminology for binary trees

In a **perfect** binary tree, each node has zero or two children and all leaves are at the same depth.

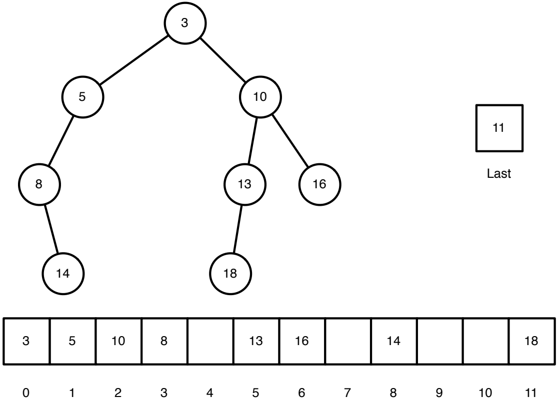
In a **complete** binary tree every level, except possibly the last, is completely filled, and all nodes on the last level are as far to the left as possible.



Contiguous implementation of a ADT Binary Tree

Data structures:

- Array storing values level by level as if all nodes were present
- Variable *Last* with the last index storing an element



Computing a sibling

Options for computing a sibling:

- Write an algorithm using existing ADT operations.
- Augment the ADT by adding a new operation.

Use existing ADT operations:

- Use *Parent* to find parent.
- Use *Left_Child* and *Right_Child* to find children of parent.
- If there is only one, return *False*.
- If there are two, return the one which is not the node itself.

Modifying the implementations for the augmented ADT

Linked implementation:

- Use *Parent* pointer to find parent.
- Use *Left* and *Right* pointers to find children of parent.
- If there is only one, return *False*.
- If there are two, return the one which is not the node itself.
- Cost is $\Theta(1)$.

Contiguous implementation:

- For node at odd index p, index of sibling is $p+1$ (if $p+1$ is at most *Last*).
- For node at even positive index p, index of sibling is $p-1$.
- Cost is $\Theta(1)$.

ADT Ordered Tree

Preconditions: For all *O* is an ordered tree, *Node* is a node in *O*, and *Data* is a data item; for *ONE_CHILD* *Index* is a nonnegative integer at most one less than the number of children of *Node*; for *ADD_LEAF* *Par* is a node in *O* or empty and *Sib* is a child of node *Par* or empty.

Postconditions: Mutation by *ADD_LEAF* (creates a new node containing *Data* to replace/add the root if *Par* is empty, as the first child of *Par* if *Sib* is empty, and otherwise as the next sibling of *Sib*).

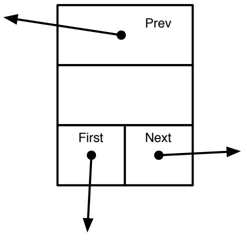
(*CREATE*, *IS_EMPTY*, *ROOT*, *VALUE*, *PARENT*, *SET_VALUE*, and *DELETE_LEAF* like in ADT Binary Tree)

Name	Returns
CHILDREN(O, Node)	all children of *Node* (Group B ADT)
ONE_CHILD(O, Node, Index)	child *Index*
ADD_LEAF(O, Par, Sib, Data)	new added node storing *Data*

Linked implementation of ADT Ordered Tree

Data structures:

- Variable *Root* pointing to root node, if any
- Nodes storing data items and three pointers *Prev* (to parent if first child or previous sibling otherwise), *First* (to first child), and *Next* (to next sibling)



Pseudocode for *Parent(O, Node)*

Use dot notation for fields inside a node in the linked structure.

if *Root(O) == Node*
    return *False*
Found $\leftarrow$ *False*
Current $\leftarrow$ *Node*
while not *Found*
    Previous $\leftarrow$ *Current.Prev*
    if *Current == Previous.First*
        Found $\leftarrow$ *True*
    else
        Current = Previous
return *Previous*

Computing the next sibling

Options for computing the next sibling:

- Write an algorithm using existing ADT operations.
- Augment the ADT by adding a new operation.

Using existing ADT operations:

- Use *Parent* to find parent.
- Use *Children* to find children.
- Scan children to determine next sibling.

Modifying the linked implementation:

- Use *Next* pointer to find next sibling.

Defining and implementing ADT Unordered Tree

ADT definition:

- Similar to ADT Ordered Tree
- Specify only parent, not sibling, when adding a node

Data structures:

- Same data structure as for ADT Ordered Tree
- Adapt algorithms to exploit fact that order of children is not significant

Returning all nodes

Options for returning all nodes:

- Write an algorithm using existing ADT operations.
- Augment the ADT by adding a new operation.

We can find all nodes by determining the root using *Root* and then repeatedly using *Children* (or *Left_Child* and *Right_Child*) to determine all descendants of the root.

Alternatively, we can use the recursive definition of a tree, in which each child of the root can be viewed as the root of a (smaller) tree. The result for the original tree will be determined using the results (obtained recursively) on the smaller trees.

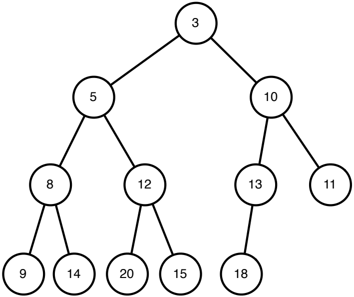
Tree traversals

A **tree traversal** is an ordering of the nodes in the tree.

- In a **level order traversal**, nodes appear in increasing order of depth.
- In a **postorder traversal**, each node appears after its children.
- In a **preorder traversal**, each node appears before its children.
- In an **inorder traversal** (only in a binary tree), for each node all nodes in the left subtree come before the node and all nodes in the right subtree come after the node.

Note: Traversals can be viewed as templates for processing (not just numbering) nodes in a given order.

Traversal example



Algorithms for traversals

Level order:

- Use *Root* to set the current node to the root.
- Use *Children* or *Left_Child* and *Right_Child* to determine children of the current node.
- Add the children to an ADT Queue.
- Repeat the process with the first node in the queue as the current node.

All other traversals:

- Create a recursive algorithm that numbers nodes starting at a given number and produces the last number used.
- For postorder, number the subtrees (in order if a binary or ordered tree), and then give the next number to the root.
- For preorder, give the first number to the root and then number subtrees (in order if a binary or ordered tree).
- For inorder, number the left subtree, give the next number to the root, then number the right subtree.

Modifying an implementation

In a linked implementation, we can **thread** nodes together by adding an extra pointer from a node to the next node in the traversal.

In a contiguous implementation, we obtain a level-order traversal by examining values in order of increasing index.

Module summary

Topics covered:

- Case study: Web page
- Trees
- Decision tree
- Data stored in nodes
- Operations for trees
- ADT Binary Tree
- Linked implementation
- Contiguous implementation
- Perfect and complete trees
- Computing siblings
- ADT Ordered Tree
- Linked implementation
- Computing the next sibling
- ADT Unordered Tree
- Tree traversals