CS 240 Fall 2024 Final Review

Question 1

Algorithm Analysis

Find a tight Θ bound on the runtime of the following algorithm:

```
def algo(n):
  i = 0
  j = n
  while i < j^2:
    i += 1
    j -= 1
```

Question 2

Priority Queues

What would be the best time complexity achievable for `insert` and `delete-max` in a priority-queue implemented with the following data structures

a) dynamic-array

b) binary heap

c) AVL tree

d) Skip list

e) Hash table implementing Cuckoo Hashing

Question 3

Tries

a) True or false, the height of a compressed trie is always in $O(\log n)$

b) Draw the binary trie, pruned trie and compressed trie containing the following keys 101, 1001, 1001000, 10011, 1110, 11111, 11100, 111110, 111101

Question 4

Hashing

a) True or false, if the load factor (α) is 1 and we're using hashing with linear probing then the next insertion is guaranteed to fail

b) True or false, if using Cuckoo hashing then insertion may take an arbitrarily long time

c) Design a data structure that has amortized expected $O(1)$ insert, search and delete and amortized $O(\log n)$ insert, search and delete

Question 5

Quadtrees

Let $S \subset \mathbb{R}^2$ such that all points in S have non-negative x and y coordinates. Assume further that $(0, 0) \in S$. Let M be the greatest distance between any 2 points in S and let m be the minimum distance between any 2 points in S. Show that if we have that $\frac{M}{m} \in O(\sqrt{n})$ then the height of the quadtree which stores the points in S will have height in $O(\log n)$

Question 6

Run Length Encoding

a) Encode the binary string "11111000001111000100000" using RLE with encoding for positive integers.
b) Is it possible that the string "110001111101" was encoded using RLE encoding for positive integers?

Question 7

LZW Encoding

A secret message was intercepted as follows.

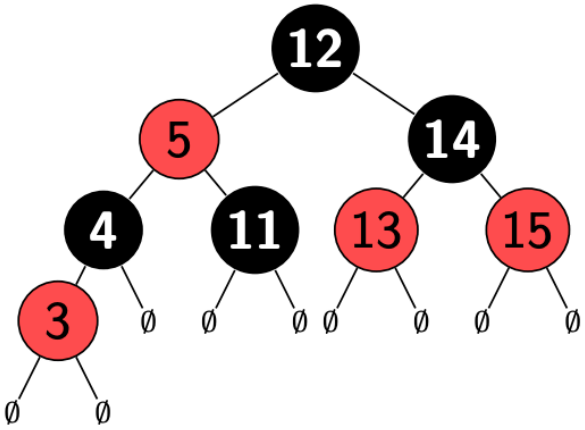
00011001 00100001 01000000 00101000 00100001 00011000 00100001 01000000 00001011 00001011

Intelligence from other sources has indicated that the message was encoded using LZW encoding with 8 bit codewords where the initial dictionary had size 64 although the original dictionary is unknown. Agent Mark did not know the original message or the original dictionary but was immediately able to tell that their intelligence was false. How was he able to determine this?

Question 8

Red-Black Tree

Convert the following Red-Black Tree into a 2-4 tree.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