October 7th 2024

# CS 240 Midterm Review

## Question 1

True or False?

a) If $\lim_{n\to\infty} \frac{f(n)}{g(n)} = e^{42}$ then $f(n) \in \Theta(g(n))$
b) If $f(n) \in O(g(n))$ then $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$
c) The height of a binary search tree with $n$ nodes is in $O(n)$.
d) All binary heaps satisfy the AVL height balance property
e) There may be undiscovered comparison based sorting algorithms that are asymptotically faster than heapsort

## Question 2

Order Notation

a) Show that $3n^2 - 8n + 2 \in \Theta(n^2)$ from first principles
b) Prove from first principles that $\frac{1}{2}n - 22 \in o(n \log(n))$
c) Prove that $f(n) \in o(g(n)) \Leftrightarrow g(n) \in \omega(f(n))$

# Question 3

Algorithm Analysis

a)
```
i = 2
x = 0
while i < n:
  for j = 1,...,n:
    for k = 1,...,j:
      x = x + 1
  i = i * i
```

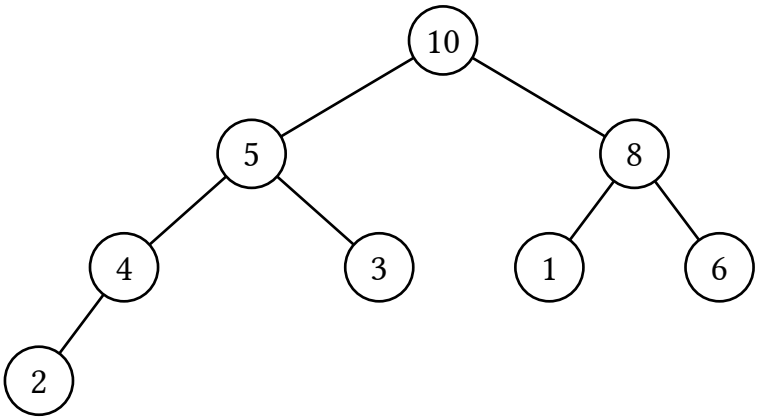
b)
```
def isSorted(A, n = A.size): #find average runtime
  for i = 0,...,n-2:
    if A[i] > A[i+1]: return False
  return True
```

c)
```
def foo(n): #find expected runtime
  if n == 0: return
  r = random(n)
  foo(r)
```

# Question 4

Binary Heaps

a) Insert 9 into the following max-heap and then delete-max




b) Describe an algorithm for merging 2 binary heaps. That is, given 2 heaps $A$ and $B$, return a new heap $C$ containing all of the elements of $A$ and $B$.

# Question 5

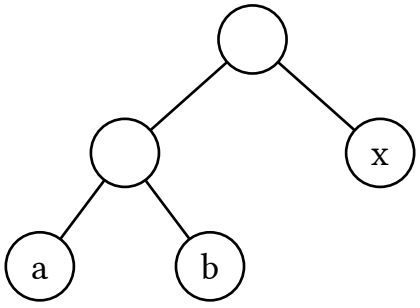
Under what circumstances can we conclude that the randomized version of an algorithm has the same expected runtime as the average runtime of the original algorithm?

# Question 6

Sorting

In real world applications, oftentimes multiple sorting algorithms are used together to achieve the best possible performance. For example, we know that quick-sort has expected runtime of $O(n \log(n))$ but it may run as slow as $O(n^2)$. One strategy to get around this is to keep track of the recursion depth while doing quick-sort and if the recursion depth is $\geq \log(n)$ at any point, the algorithm switches to heap-sort. Prove that this algorithm has worst case runtime in $O(n \log(n))$.

# Question 7




If the following tree is an AVL tree, where $x, a,$ and $b$ represent subtrees, then if $x$ has height $5$ and $b$ has height $2$ what are the possible values for the height of $a$?