

University of Waterloo

CS240 - Fall 2025

Assignment 1

Due Date: Tuesday September 16 at 5:00pm

Please follow the guidelines for submission on the course webpage.

<https://student.cs.uwaterloo.ca/~cs240/f25/assignments.phtml#guidelines>

Each question must be submitted individually to Crowdmark. Submit early and often.

Grace period: submissions made before 7:59PM on September 16 will be accepted without penalty. Your last submission will be graded. Please note that submissions made after 7:59PM **will not be graded** and may only be reviewed for feedback.

All logs are base 2. There are 63 marks available; the assignment will be marked out of 60.

Problem 1 [3+3+3+3+3=15 marks]

Provide a complete proof of the following statements from first principles (i.e., using the original definitions of order notation).

- | | |
|--|------------------------------------|
| a) $12n^3 + 11n^2 + 10 \in O(n^3)$ | d) $1000n \in o(n \log n)$ |
| b) $12n^3 + 11n^2 + 10 \in \Omega(n^3)$ | e) $n^n \in \omega(n^{20})$ |
| c) $12n^3 + 11n^2 + 10 \in \Theta(n^3)$ | |

Problem 2 [4+4=8 marks]

For each pair of the following functions, fill in the correct asymptotic notation among Θ , o , and ω in the statement $f(n) \in \square(g(n))$. Provide a brief justification of your answers. In your justification you may use any relationship or technique that is described in class.

- a)** $f(n) = \sqrt{n}$ versus $g(n) = (\log n)^2$
- b)** $f(n) = n^3(5 + 2 \cos 2n)$ versus $g(n) = 4n^3 + 3n^2 + 5n$

Problem 3 [6+6=12 marks]

Prove or disprove each of the following statements. To prove a statement, you should provide a formal proof that is based on the definitions of the order notations. To disprove a statement, you can either provide a counter example and explain it or provide a formal proof. All functions are positive functions.

a) $f(n) \notin o(g(n))$ and $f(n) \notin \omega(g(n)) \Rightarrow f(n) \in \Theta(g(n))$

b) $\min(f(n), g(n)) \in \Theta\left(\frac{f(n)g(n)}{f(n)+g(n)}\right)$

Problem 4 [6 marks]

Suppose n is a power of two and θ is a parameter in the range $2 \leq \theta \leq 3$. Derive an exact closed form for the sum

$$f(n) := \sum_{i=0}^{\log_2 n} 4^i \left(\frac{n}{2^i}\right)^\theta$$

in terms of n and θ . *Hints:* Re-write the formula as a geometric series. Treat $\theta = 2$ as a special case.

Problem 5 [2+2+4+4=12 marks]

Consider the following procedure.

```
pre: n is a positive integer
pre: v[1..n] is a binary vector of length n,
      i.e., each entry is either 0 or 1
foo(v,n)
1.  i := 1;
2.  while i<=n and v[i]=0 do
3.      i := i+1
4.  od;
5.  for j from 1 to i do
6.      print("Hello world!")
7.  od;
```

- a) How many possible input vectors are there of size n ?
- b) What is the worst case number of calls to print? Give an exact formula in terms of n and justify your answer by giving an example of a worst case input of size n . *Note:* This is standard pseudo-code, so the for-loop is inclusive of i . E.g., if $i = 1$ then the loop executes once.
- c) For $i \in \{1, 2, \dots, n\}$, let S_i denote the subset of inputs of size n for which the number of calls to print is i . Describe what an element of S_i looks like, and derive an expression for $|S_i|$, the number of elements of S_i .
- d) What is the average case number of calls to print? Derive an exact closed form formula in terms of n .

Problem 6 [5 marks]

Prove that the following code fragment will always terminate.

```
s := 3*n // n is an integer
while (s>0)
    if (s is even)
        s := floor(s/4)
    else
        s := 2*s
```

Problem 7 [5 marks]

Analyze the following piece of pseudo-code and give a Θ bound on the running time as a function of n . Show your work. A formal proof is not required, but you should justify your answer.

<pre>1. <i>mystery</i> \leftarrow 0 2. for $i \leftarrow 1$ to $3n$ do 3. <i>mystery</i> \leftarrow <i>mystery</i> \times 4 4. for $j \leftarrow 1388$ to 2010 do 5. for $k \leftarrow 4i$ to $6i$ do 6. <i>mystery</i> \leftarrow <i>mystery</i> + k</pre>
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