University of Waterloo
CS240 Spring 2022
Assignment 1 Post-Mortem

This document goes over common errors and general student performance on the assignment questions. We put this together using feedback from the graders once they are done marking. It is meant to be used as a resource to understand what we look at while marking and some common areas where students can improve in.

[General]

- A few students submitted blurry pictures of their handwritten answers. In accordance with the course policy, one mark was deducted from each question for difficult-to-read submissions.
- Properties that were not used or derived in course material must be proved before being used.
- Some proofs were not sufficiently detailed, or many steps were skipped along the way. Your work should give a clear idea of what is being done, with justification for non-obvious steps.

Question 1 [3+3+4=10 marks]

- When proving order notation from first principles, explicit values for c and n_0 that satisfy the relationship must be given.
- The values c and n_0 should be derived rather than guessing and stating them first and then proving they are valid.

Question 2 [3+3+4=10 marks]

- Many students forgot to mention what rule or theorem (e.g. l'Hopital's rule) they used in their proof and instead jumped to the next step without any justification.
- Many students forgot that the derivative of $\log_2(n)$ is $\frac{1}{n \log 2}$ rather than $\frac{1}{n}$ (the derivative of $\ln(n)$).
- While using the limit rule, a few students calculated $\lim_{n\to\infty} \frac{g(n)}{f(n)}$ instead of $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ and thus reached the incorrect answer.

Question 3 [4+4+4+4+4=20 marks]

- For a), some students forgot to explicitly state a value of n_0 when proving that $f(n) \in \Omega(g(n))$.
- For b), some students attempted to prove the false statement using the zero function(s) $f(n) = g(n) = 0$, but the question required $f(n)$ and $g(n)$ to map positive integers to positive integers and 0 is not a positive integer.
- For c), some students attempted to prove the (false) statement using the values of c and n_0 from the definition of $f(n) \in O(g(n))$ and stating without proof that these values work for $2^{f(n)}$ and $2^{g(n)}$.
- For d). some students attempted to prove the (false) statement by using the limit rule and incorrectly differentiating $\log(n)^{\log(n)}$.

Question 4 [2+4=6 marks]

- For a), many students used the value $n_0 = 0$ in their proof, but the definition of $g(n) \in O'(f(n))$ applies for all $n > 0$ and $0 \not> 0$. Thus, a value of n_0 that is atrictly larger than 0 would be an appropriate choice.
- For b), some proofs were too informal to be considered correct. At minumum, a correct proof for this question should define constants c (based on the first few terms of the sequence $\frac{g(n)}{f(n)}$ or $\frac{f(n)}{g(n))}$) and a value for n_0 such that $g(n) \leq cf(n))$ for all $n \geq n_0$.

Question 5 [4+4+5=13 marks]

- A few students did not provide a Theta (Θ) bound like the question asked for and instead proved something else (e.g. O-notation). Remember that both upper and lower bounds need to be proved to get a tight Θ-bound.
- For a), a few students incorrectly wrote $\sum_{i=1}^{n} i^2 \in n^2$ and thus had a Θ-bound of ‘ : $xn^2 \log n$ instead of $n^3 \log n$.
- For b), some students deduced that i with initial value 2 was squared $\lceil \log n \rceil$ times instead of $\lceil \log \log n \rceil$ times by incorrectly stating that $2^k \geq n$ instead of $2^{2^k} \geq n$, where k is the number of times i is squared in the loop.
- For a), many students did not state that the children are ordered from left-to-right in the array. An ordering is necessary for the array structure to be unambiguous.
- For b), some students only proved an upper bound. Remember that both upper and lower bounds need to be proved to get a tight bound.

- For c), many students reached a final answer of $\Theta(n^3)$ or $\Theta(n^4)$ instead of $\Theta(n^3 \log n)$. Note that the partial sum of the harmonic series is in $\Theta(\log n)$; this can be found in the Useful Sums slide in the course lectures.