

## Tutorial 01: May 22

1.  $\Theta$  and  $o$ -notation

- (a) Prove from first principles that  $n^3 \in \Theta(4n^3 - 3n^2 + 2n - 1)$ .  
 (b) Prove from first principles that  $2000n^2 \in o(n^n)$ .

2.  $O$  and  $\omega$  Prove (from first principles) or disprove (by providing a counterexample) the following claim.

If  $f(n) \in O(h_1(n))$  and  $g(n) \in \omega(h_2(n))$ , then  $\frac{f(n)}{g(n)} \in o\left(\frac{h_1(n)}{h_2(n)}\right)$ , where  $f(n), g(n), h_1(n)$  are all positive for all  $n \geq 0$ .

3. **Factorials** Prove from first principles that  $\log(n!) \in \Theta(n \log n)$ .4. **Parity** Define

$$f(n) = \begin{cases} n^2 & \text{if } n \text{ is even} \\ n & \text{if } n \text{ is odd} \end{cases}$$

- (a) Is  $f(n) \in O(n^2)$ ?  
 (b) Is  $f(n) \in \Omega(n^2)$ ?  
 (c) Is  $f(n) \in \Theta(n^2)$ ?  
 (d) Is  $f(n) \in o(n^2)$ ?

**Bonus:** Construct a function  $g(n)$  such that  $g(n) \in o(n^2)$  but  $g(n) \notin \Theta(n)$ .

**Bonus\*:** Construct a *continuous, strictly increasing* function  $f(x)$  such that  $f(x) \notin O(x^2)$  and  $f(x) \notin \Omega(x^2)$ .

5. **Loop analysis** Provide a tight  $\Theta$  bound on the following algorithm as a function of  $n$ :

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**Algorithm 1:** Iterative pseudocode

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1  $k \leftarrow 1$ 
2 for  $i$  from 1 to  $n$  do
3    $j \leftarrow 0$ 
4   while  $j \leq n$  do
5      $j \leftarrow j + k$ 
6   end
7    $k \leftarrow 2k$ 
8 end

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