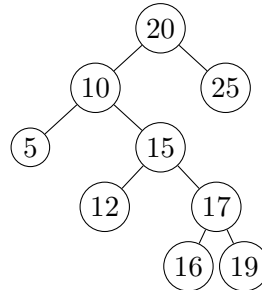


There are fewer problems this week due to Tuesday's midterm. If time allows and marking is progressing well, I may discuss some common errors in the midterm questions.

Q1. Splay trees. Given the following splay tree S , calculate its potential using the potential function

$$\Phi(i) := \sum_{v \in S} \log n_v^{(i)},$$

where $n_v^{(i)}$ is the number of nodes in the subtree rooted at v after i operations, including v itself. Insert the key 18. Calculate the new potential. Verify that the difference between the potential difference is less than $4 \log n - 2R + 2$, where R is the number of rotations.



Q2. Interpolation Search Practice

In the array below, *interpolation-search*(70) finds the item at the first probe that uses the formula. What is the smallest possible integer at XX ?

Algorithm 1: *interpolation-search*(A, n, k)

```

1  $\ell \leftarrow 0, r \leftarrow n - 1$  ;
2 while ( $\ell \leq r$ ) do
3   if ( $k < A[\ell]$ ) then return “not found, would be between indices  $\ell - 1$  and  $\ell$ ”;
4   if ( $k > A[r]$ ) then return “not found, would be between indices  $r$  and  $r + 1$ ”;
5   if ( $k = A[r]$ ) then return “found at index  $r$ ”;
6    $m \leftarrow \ell + \lceil \frac{k - A[\ell]}{A[r] - A[\ell]} \cdot (r - \ell - 1) \rceil$  ;
7   if ( $A[m] = k$ ) then return “found at index  $m$ ”;
8   else if ( $A[m] < k$ ) then  $\ell \leftarrow m + 1$ ;
9   else  $r \leftarrow m - 1$ ;
10 end

```

0	1	2	3	4	5	6	7	8	9	10
XX	57	64	70	72	77	81	92	93	99	100

Q3. Interpolation Search Runtime Suppose we have an array A with $A[i] = f(i)$. What is the worst-case runtime for interpolation search on A with the following functions?

(a) $f(i) = i$

(b) $f(i) = 2^i$

(c) (Bonus) $f(i) = i^2$

(d) (Bonus) $f(i) = \log i$ (note that the array starts at 1).

Note that the bonus are mostly provided as a fun thought - it could be very ugly to prove, and solutions will not be provided.

Q4. Compressed trie.

Let T be a compressed trie of n strings from an alphabet Σ , which we assume does not contain the end-of-character symbol.

1. What is the maximum height of T ?
2. What is the minimum height of T ?

Give exact expressions (not order notation) in terms of n and $|\Sigma|$. If the answer depends on which of n and $|\Sigma|$ is greater, give answers for each scenario.

In the next 5 empty pages, mentally insert the midterm...

