CS240E S26	Tutorial 9	July 10

Q1. Range tree space. Prove or disprove: for any set of n points in general position, the range tree uses $\Omega(n \log n)$ space.

Q2. kd-tree. Create a set of n points and a range-query such that doing the range-query on the kd-tree of the points requires $\Omega(\sqrt{n})$ boundary nodes.

Q3. Quad-tree.

(a) For an arbitrary n, construct a set of points such that the quad-tree has at least n nodes, and give a range-search query such that all nodes are visited, and not a single point gets returned.

(b) Assume that T is a quad-tree with at least two points such that during some range-search, there is at least one outside node and at least one inside-node (the example from Module 8, slide 11 satisfies this). What is the minimum possible height of T?

The example has height 3, so the question is whether height 3 is always required, or whether this could also happen with height 2 or even height 1?

Q4. Point in polygon. Suppose you are given a polygon by its n vertices, (x_i, y_i).

(a) Given a point (x, y), give an algorithm that determines whether (x, y) is inside or outside the polygon in $O(n)$ time.

(b) Suppose you have multiple points. How would you effectively test for whether they are inside of this polygon?

Not-A-Q5. Convex Hull Algorithms. In this "question", we will see some common algorithms for finding the convex hull of a set of points.

Q6. Convex Hull Optimization. (https://codeforces.com/gym/103536/problem/B)

Peter has found a litter of N kittens lying around in an alley somewhere and wants to give them up for adoption. However, kittens are trouble, they knock over your ornaments, dirty your house, etc. Luckily, Peter has found a way to measure how much trouble a group of kittens will cause.

Peter first identifies each kitten with two attributes, a naughtiness level A_i and a cuteness level B_i. If he sends a particular group of cats to one owner, the trouble they will cause is given by the formula $\max(A_i) \cdot \max(B_i)$ over the group of kittens.

Peter wants to minimize the total trouble caused by these kittens by grouping them optimally and sending them to their new owners in those groups. Help Peter find out what this minimum total trouble is.

Note: the intended time complexity is $O(n \log n)$.