CS240E W25	Tutorial 1	Jan. 10

Overview

- Asymptotic notation review
- Two approaches to $n \in \omega\left(2^{\sqrt{\lg n}}\right)$
- Little-o characterization
- Big-O with divisibility assumption
- Runtime analysis example
- Bounds with integration

Problems

Q1. Show that $n \in \omega\left(2^{\sqrt{\lg n}}\right)$ using the definition.

Q2. Let $f(x)$ be a positive monotone function with domain $\mathbb{R}^+$. Assume that $f(x) \leq x$ for all even integers x. Show that $f(x) \in O(x)$.

Q3. Let $f(n), g(n)$ be eventually positive. In lecture we saw that if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$ then $f(n) \in o(g(n))$. Prove the converse.

Q4. Give a tight bound on the runtime of the following algorithm as a function of n.

```
k = 1
for(i = 1; i <= n; ++i):
    j = 0
    while j <= n:
        j += k
    k *= 2
```

Additional problems

Q5. Give an exact bound on the n-th Harmonic number $H_n := \sum_{k=1}^{n} 1/k$. Note: this question requires bounds with integration.

Q6. Assuming any appropriate base case, give asymptotic upper and lower bounds (make them as tight as you can) for $T(n)$ if,

(a) $T(n) = 3T(n/3) + n/\log_3 n$

(b) $T(n) = T(n-1) + 1/n$

(c) $T(n) = \sqrt{n}T(\sqrt{n}) + n$