CS240E W25	**Tutorial 5**	Feb. 7

Overview

- Skip lists
- Splay Trees
- Static Ordering for Biased Searches
- Counting Trees

Problems

Q1. Static Ordering.

Let A be an unordered array with n distinct items $k_0, ..., k_{n-1}$. Give an asymptotically tight (Θ) bound on the expected access cost if you put A in the optimal static order for the following probability distributions:

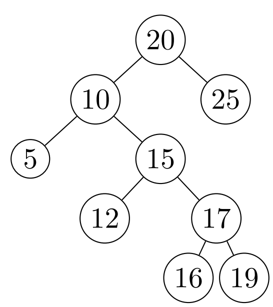
1. $p_i = \frac{1}{n}$ for $0 \leq i \leq n-1$
2. $p_i = \frac{1}{2^{i+1}}$, for $0 \leq i \leq n-2$, $p_{n-1} = 1 - \sum_{i=0}^{n-2} p_i = \frac{1}{2^{n-1}}$

Q2. Splay Trees.

Given the following splay tree S, calculate its potential using the potential function

$$\Phi(i) := \sum_{v \in S} \log n_v^{(i)},$$

where $n_v^{(i)}$ is the number of nodes in the subtree rooted at v after i operations, including v itself. Insert the key 18. Calculate the new potential. Verify that the potential difference is less than $4 \log n - 2R + 2$, where R is the number of rotations.



Q3. Skip Lists.

Insert the numbers 12, 11, 13, 10, and 20 into an empty skip-list using the sequence of coin flips HHTHTHTTHHHT (i.e., every time we go to do a coin flip we take the first item out of this list). Then delete the keys 13 and 20.

Q4. Counting Trees.

How many binary trees with n nodes are there, as a formula in terms of n? Find a recurrence relation.

(There is also a closed-form for this recurrence relation, but deriving it is outside the scope of this course.)