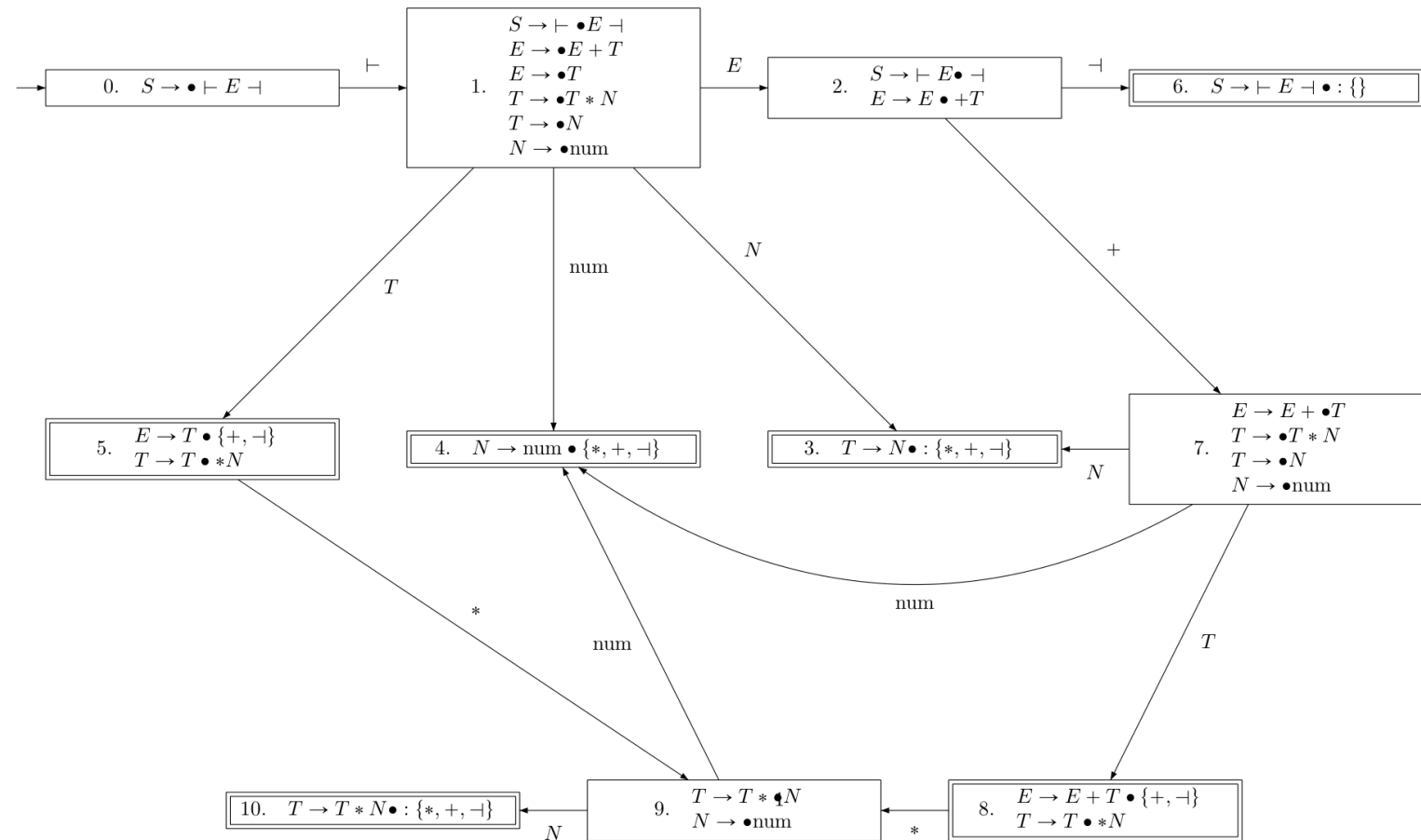# Bottom-Up Parsing:
SLR(1) and LR(1)

SLR(1): Using Follow Sets to Resolve Conflicts

- The idea of SLR(1) is to use the same DFA construction as LR(0), but every reducible item is "tagged" with the Follow set of the LHS.

- If you are in a reduce state, and the lookahead (first symbol of unread input) is in the "tag" of a reducible item, reduce using that rule.
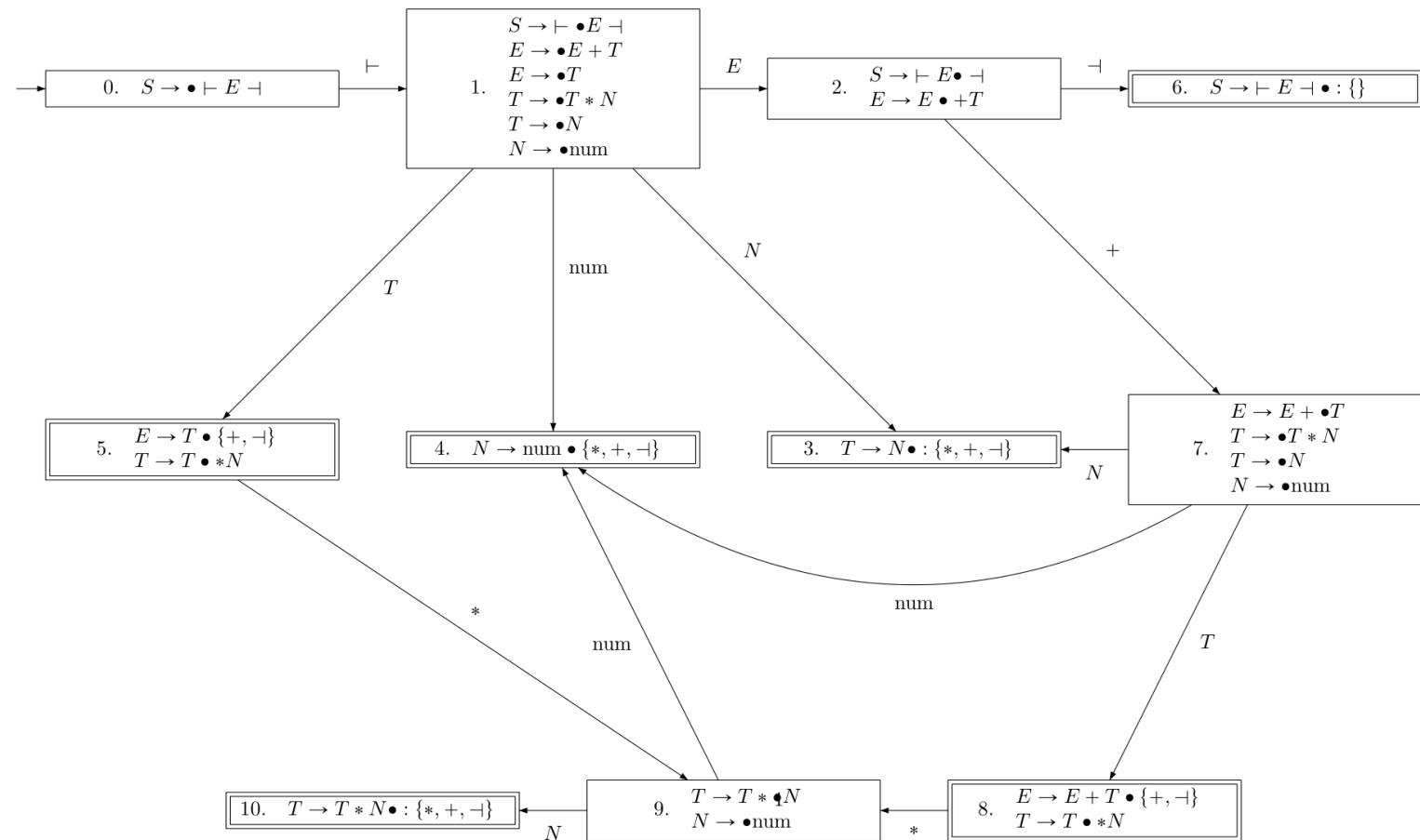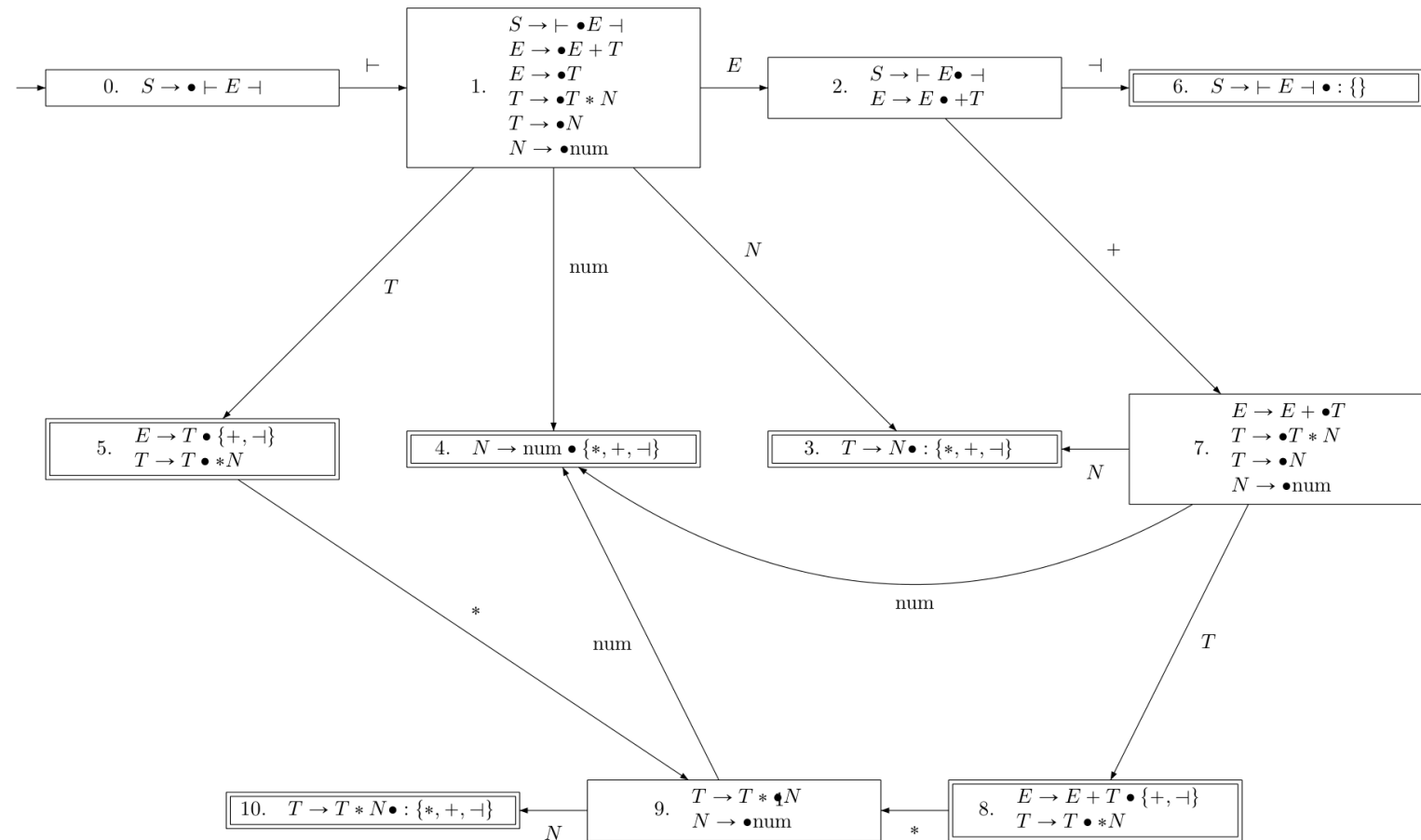
- Otherwise, shift.

SLR(1): Using Follow Sets to Resolve Conflicts

- The idea of SLR(1) is to use the same DFA construction as LR(0), but every reducible item is "tagged" with the Follow set of the LHS.

- Shift-Reduce conflicts are resolved if the symbol to be shifted is not in the Follow set for the reducible item.

- Reduce-Reduce conflicts are resolved if the Follow sets don't overlap.

SLR(1): Using Follow Sets to Resolve Conflicts

- For example, if the reduction sequence is:
  ⊢ **T**

- If the next symbol of unread input is + or ⊣ we reduce by E → T.

- Otherwise, we shift.

- If the rest of input is
  ⊣
  or
  + T ⊣
  reducing is correct.

- If it's * N ⊣ then shifting is correct.

The LR(0) Parsing Algorithm: Pseudocode

```
initialize symbolStack to an empty stack
initialize stateStack with the initial state of the parsing DFA
while input is not empty:
  a = first symbol of input
  (Reduce until we are no longer in a reduce state)
  while stateStack.top contains a reducible item [A→α•]:
    len = length of α                      (number of symbols on right-hand side)
    pop len symbols from symbolStack       (pop the right-hand side)
    push A to symbolStack                  (push the left-hand side)
    pop len states from stateStack         (backtrack in the DFA)
    let newState be obtained by taking the transition from stateStack.top on A
    push newState to stateStack            (state stack is again synchronized with symbol stack)
  (Once we can no longer reduce, shift a symbol from input)
  if there is a transition from stateStack.top on a to newState:
    push a to symbolStack                  (push the symbol-to-shift)
    push newState to stateStack            (keep the state stack synchronized)
    consume a from input                   (read and remove first symbol from input)
  else
    ERROR                                  (no transition on input symbol, parse failed)
```

The **LR(1)** Parsing Algorithm: Pseudocode

```
initialize symbolStack to an empty stack                          The only difference!
initialize stateStack with the initial state of the parsing DFA
while input is not empty:
  a = first symbol of input
  (Reduce until we are no longer in a reduce state)                          ↓
  while stateStack.top contains a reducible item [A→α•] with a in the lookahead tag:
    len = length of α                    (number of symbols on right-hand side)
    pop len symbols from symbolStack     (pop the right-hand side)
    push A to symbolStack                (push the left-hand side)
    pop len states from stateStack       (backtrack in the DFA)
    let newState be obtained by taking the transition from stateStack.top on A
    push newState to stateStack          (state stack is again synchronized with symbol stack)
  (Once we can no longer reduce, shift a symbol from input)
  if there is a transition from stateStack.top on a to newState:
    push a to symbolStack                (push the symbol-to-shift)
    push newState to stateStack          (keep the state stack synchronized)
    consume a from input                 (read and remove first symbol from input)
  else
    ERROR                                (no transition on input symbol, parse failed)
```

SLR(1) vs. LR(1)

- The LR(1) parsing algorithm can be used with any kind of parsing DFA that has "lookahead tags".

- The SLR(1) DFA uses Follow sets as lookahead tags.

- The term **LR(1) DFA** refers to a more complex construction (not covered in this course) where only a subset of the Follow set is used.
  - The LR(1) DFA resolves more LR(0) conflicts than the SLR(1) DFA, but the number of states can be exponentially larger than the SLR(1) DFA.

- There is also something called LALR(1) ("Lookahead LR(1)") which is a compromise and is popular in practice. It resolves more conflicts than SLR(1), and uses less states than LR(1) but resolves fewer conflicts.

Building a Parse Tree

- The pseudocode on the previous slides doesn't actually produce any result. It either runs to completion, or produces an error.

- To make it produce a **derivation**, we could modify it to output the reduce rule every time we do a reduce step.
    - The derivation would be in reverse order.

- A better option is to make it produce a **parse tree**.

- The idea is to replace the symbol stack with a **tree stack**.
    - When shifting, we add leaf nodes corresponding to the shifted terminal.
    - When reducing, we pop tree nodes corresponding to the rule RHS, make them children of a new node with the LHS, and push this new tree.

⊢ num * num ⊣

State Stack: 0

Tree Stack Top

Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1

Tree Stack Top

$$0. \quad S \rightarrow \bullet \vdash E \dashv$$

⊢

$$
\begin{aligned}
S &\rightarrow \vdash \bullet E \dashv \\
E &\rightarrow \bullet E + T \\
E &\rightarrow \bullet T \\
T &\rightarrow \bullet T * N \\
T &\rightarrow \bullet N \\
N &\rightarrow \bullet \text{num}
\end{aligned}
$$
⭐ 1.

E

$$2. \quad \begin{aligned} S &\rightarrow \vdash E \bullet \dashv \\ E &\rightarrow E \bullet +T \end{aligned}$$

⊣

$$6. \quad S \rightarrow \vdash E \dashv \bullet : \{\}$$

T

num

N

+

$$5. \quad \begin{aligned} E &\rightarrow T \bullet \{+, \dashv\} \\ T &\rightarrow T \bullet *N \end{aligned}$$

$$4. \quad N \rightarrow \text{num} \bullet \{*, +, \dashv\}$$

$$3. \quad T \rightarrow N \bullet : \{*, +, \dashv\}$$

$$7. \quad \begin{aligned} E &\rightarrow E + \bullet T \\ T &\rightarrow \bullet T * N \\ T &\rightarrow \bullet N \\ N &\rightarrow \bullet \text{num} \end{aligned}$$

N

num

*

num

T

$$10. \quad T \rightarrow T * N \bullet : \{*, +, \dashv\}$$

N

$$9. \quad \begin{aligned} T &\rightarrow T * \bullet N \\ N &\rightarrow \bullet \text{num} \end{aligned}$$

*

$$8. \quad \begin{aligned} E &\rightarrow E + T \bullet \{+, \dashv\} \\ T &\rightarrow T \bullet *N \end{aligned}$$

⊢

Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1 4

Tree Stack Top

$0.\quad S \to \bullet \vdash E \dashv$

$1.$
$S \to \vdash \bullet E \dashv$
$E \to \bullet E + T$
$E \to \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet \text{num}$

$\vdash$

E

$2.$
$S \to \vdash E \bullet \dashv$
$E \to E \bullet + T$

$\dashv$

$6.\quad S \to \vdash E \dashv \bullet : \{\}$

T

num

N

$+$

$5.$
$E \to T \bullet \{+, \dashv\}$
$T \to T \bullet *N$

$4.\quad N \to \text{num} \bullet \{*, +, \dashv\}$

$3.\quad T \to N \bullet : \{*, +, \dashv\}$

$7.$
$E \to E + \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet \text{num}$

N

num

*

num

num

T

$10.\quad T \to T * N \bullet : \{*, +, \dashv\}$

N

$9.$
$T \to T * \bullet N$
$N \to \bullet \text{num}$

*

$8.$
$E \to E + T \bullet \{+, \dashv\}$
$T \to T \bullet *N$

num

⊢

Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1

Tree Stack Top

$0. \quad S \rightarrow \bullet \vdash E \dashv$

$\vdash$

⭐ 1.
$S \rightarrow \vdash \bullet E \dashv$
$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet \text{num}$

E

2.
$S \rightarrow \vdash E \bullet \dashv$
$E \rightarrow E \bullet + T$

$\dashv$

$6. \quad S \rightarrow \vdash E \dashv \bullet : \{\}$

T

N

num

N

$+$

5.
$E \rightarrow T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

$4. \quad N \rightarrow \text{num} \bullet \{*, +, \dashv\}$

$3. \quad T \rightarrow N \bullet : \{*, +, \dashv\}$

7.
$E \rightarrow E + \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet \text{num}$

N

num

$*$

num

T

$10. \quad T \rightarrow T * N \bullet : \{*, +, \dashv\}$

N

9.
$T \rightarrow T * \bullet N$
$N \rightarrow \bullet \text{num}$

$*$

8.
$E \rightarrow E + T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

Reducing by N → num:
Pop num / 4

num

⊢

Tree Stack Bottom

⊢ num * num ⊣

Tree Stack Top

$$0. \quad S \to \bullet \vdash E \dashv$$

$$S \to \vdash \bullet E \dashv$$
$$E \to \bullet E + T$$
$$E \to \bullet T$$
$$1. \quad T \to \bullet T * N$$
$$T \to \bullet N$$
$$N \to \bullet \text{num}$$

$$2. \quad S \to \vdash E \bullet \dashv$$
$$E \to E \bullet + T$$

$$6. \quad S \to \vdash E \dashv \bullet : \{\}$$

$$5. \quad E \to T \bullet \{+, \dashv\}$$
$$T \to T \bullet * N$$

$$4. \quad N \to \text{num} \bullet \{*, +, \dashv\}$$

$$3. \quad T \to N \bullet : \{*, +, \dashv\}$$

$$E \to E + \bullet T$$
$$T \to \bullet T * N$$
$$7. \quad T \to \bullet N$$
$$N \to \bullet \text{num}$$

$$10. \quad T \to T * N \bullet : \{*, +, \dashv\}$$

$$9. \quad T \to T * \bullet N$$
$$N \to \bullet \text{num}$$

$$8. \quad E \to E + T \bullet \{+, \dashv\}$$
$$T \to T \bullet * N$$

N

num

Reducing by N → num:
Create node for N

⊢

Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1

Tree Stack Top

0. $S \to \bullet \vdash E \dashv$

⊢

1. $S \to \vdash \bullet E \dashv$
$E \to \bullet E + T$
$E \to \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet \text{num}$

E

2. $S \to \vdash E \bullet \dashv$
$E \to E \bullet +T$

⊣

6. $S \to \vdash E \dashv \bullet : \{\}$

5. $E \to T \bullet \{+, \dashv\}$
$T \to T \bullet *N$

4. $N \to \text{num} \bullet \{*, +, \dashv\}$

3. $T \to N \bullet : \{*, +, \dashv\}$

7. $E \to E + \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet \text{num}$

T

num

N

+

N

num

*

num

T

10. $T \to T * N \bullet : \{*, +, \dashv\}$

N

9. $T \to T * \bullet N$
$N \to \bullet \text{num}$

*

8. $E \to E + T \bullet \{+, \dashv\}$
$T \to T \bullet *N$

T

N

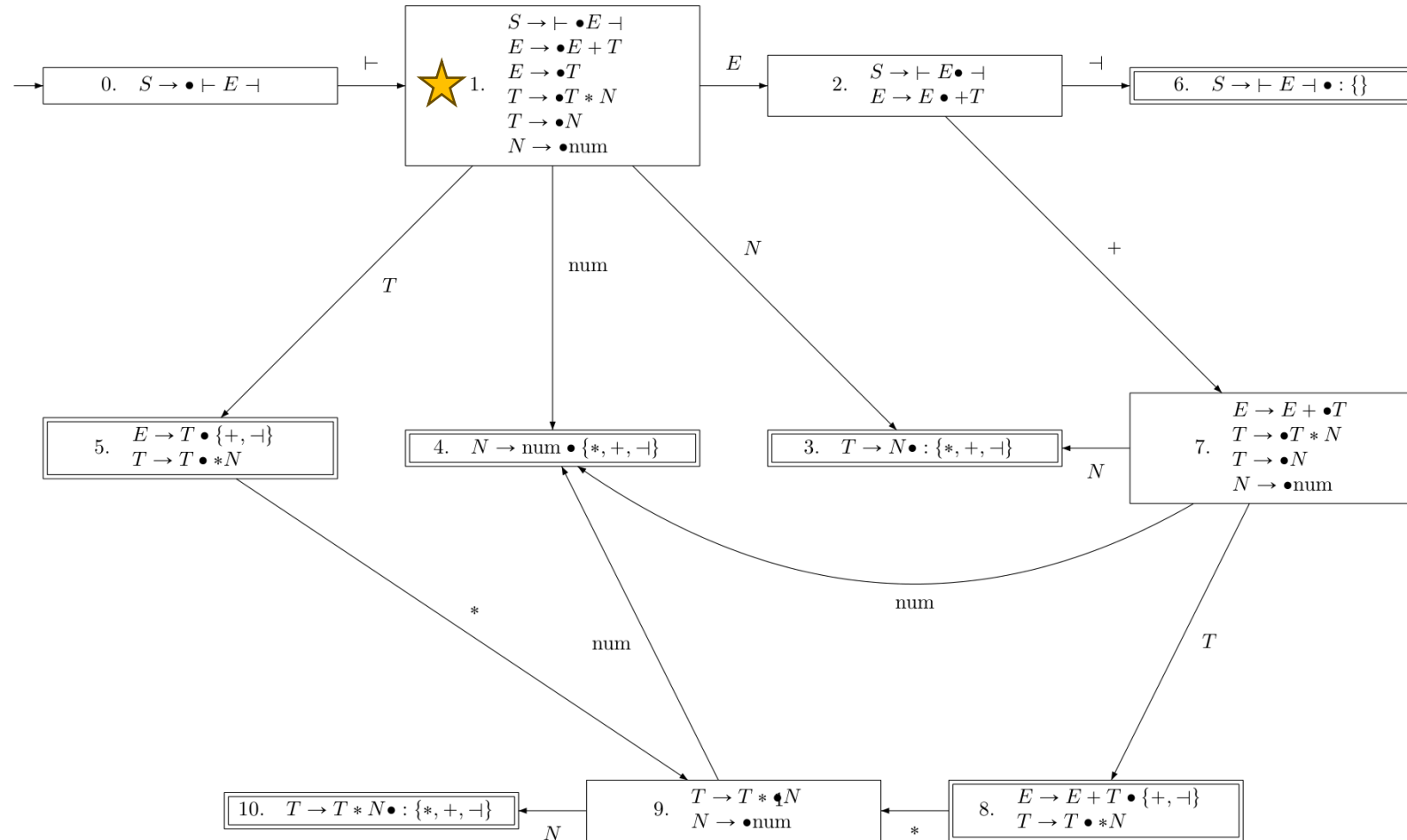Reducing by N → num:
Add num node as child

⊢

Tree Stack Bottom

$\vdash$ num $*$ num $\dashv$

State Stack: 0 1 3

Tree Stack Top

0. $S \to \bullet \vdash E \dashv$

$\vdash$

1. $S \to \vdash \bullet E \dashv$
   $E \to \bullet E + T$
   $E \to \bullet T$
   $T \to \bullet T * N$
   $T \to \bullet N$
   $N \to \bullet$num

E

2. $S \to \vdash E \bullet \dashv$
   $E \to E \bullet + T$

$\dashv$

6. $S \to \vdash E \dashv \bullet : \{\}$

T

N

num

$+$

5. $E \to T \bullet \{+, \dashv\}$
   $T \to T \bullet * N$

4. $N \to$ num $\bullet \{*, +, \dashv\}$

3. $T \to N \bullet : \{*, +, \dashv\}$

7. $E \to E + \bullet T$
   $T \to \bullet T * N$
   $T \to \bullet N$
   $N \to \bullet$num

N

Reducing by N → num:
Push N / 3

N

num

$\vdash$

$*$

num

num

T

10. $T \to T * N \bullet : \{*, +, \dashv\}$

N

9. $T \to T * \bullet N$
   $N \to \bullet$num

$*$

8. $E \to E + T \bullet \{+, \dashv\}$
   $T \to T \bullet * N$

Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1 3

Tree Stack Top

$0. \quad S \to \bullet \vdash E \dashv$

$1.$
$S \to \vdash \bullet E \dashv$
$E \to \bullet E + T$
$E \to \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet \text{num}$

$2. \quad$
$S \to \vdash E \bullet \dashv$
$E \to E \bullet + T$

$6. \quad S \to \vdash E \dashv \bullet : \{\}$

$5. \quad$
$E \to T \bullet \{+, \dashv\}$
$T \to T \bullet * N$

$4. \quad N \to \text{num} \bullet \{*, +, \dashv\}$

$3. \quad T \to N \bullet : \{*, +, \dashv\}$

$7. \quad$
$E \to E + \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet \text{num}$

$10. \quad T \to T * N \bullet : \{*, +, \dashv\}$

$9. \quad$
$T \to T * \bullet N$
$N \to \bullet \text{num}$

$8. \quad$
$E \to E + T \bullet \{+, \dashv\}$
$T \to T \bullet * N$

N

num

⊢

Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1

Tree Stack Top

0. $S \to \bullet \vdash E \dashv$

⊢

1. ★
$S \to \vdash \bullet E \dashv$
$E \to \bullet E + T$
$E \to \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet\text{num}$

E

2. $S \to \vdash E \bullet \dashv$
$E \to E \bullet + T$

⊣

6. $S \to \vdash E \dashv \bullet : \{\}$

T

num

N

+

5. $E \to T \bullet \{+, \dashv\}$
$T \to T \bullet * N$

4. $N \to \text{num} \bullet \{*, +, \dashv\}$

3. $T \to N \bullet : \{*, +, \dashv\}$

7. $E \to E + \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet\text{num}$

N

num

*

num

num

T

Reducing by T → N:
Pop N / 3

N

num

10. $T \to T * N \bullet : \{*, +, \dashv\}$

N

9. $T \to T * \bullet N$
$N \to \bullet\text{num}$

*

8. $E \to E + T \bullet \{+, \dashv\}$
$T \to T \bullet * N$

⊢

Tree Stack Bottom

⊢ num * num ⊣

Tree Stack Top

0. $S \to \bullet \vdash E \dashv$

⊢

1. ★
$S \to \vdash \bullet E \dashv$
$E \to \bullet E + T$
$E \to \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet \text{num}$

E

2. $S \to \vdash E \bullet \dashv$
$E \to E \bullet +T$

⊣

6. $S \to \vdash E \dashv \bullet : \{\}$

T

num

N

$+$

5. $E \to T \bullet \{+, \dashv\}$
$T \to T \bullet *N$

4. $N \to \text{num} \bullet \{*, +, \dashv\}$

3. $T \to N \bullet : \{*, +, \dashv\}$

7. $E \to E + \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet \text{num}$

N

num

Reducing by T → N:
Create node for T

*

num

num

T

10. $T \to T * N \bullet : \{*, +, \dashv\}$

N

9. $T \to T * \bullet N$
$N \to \bullet \text{num}$

*
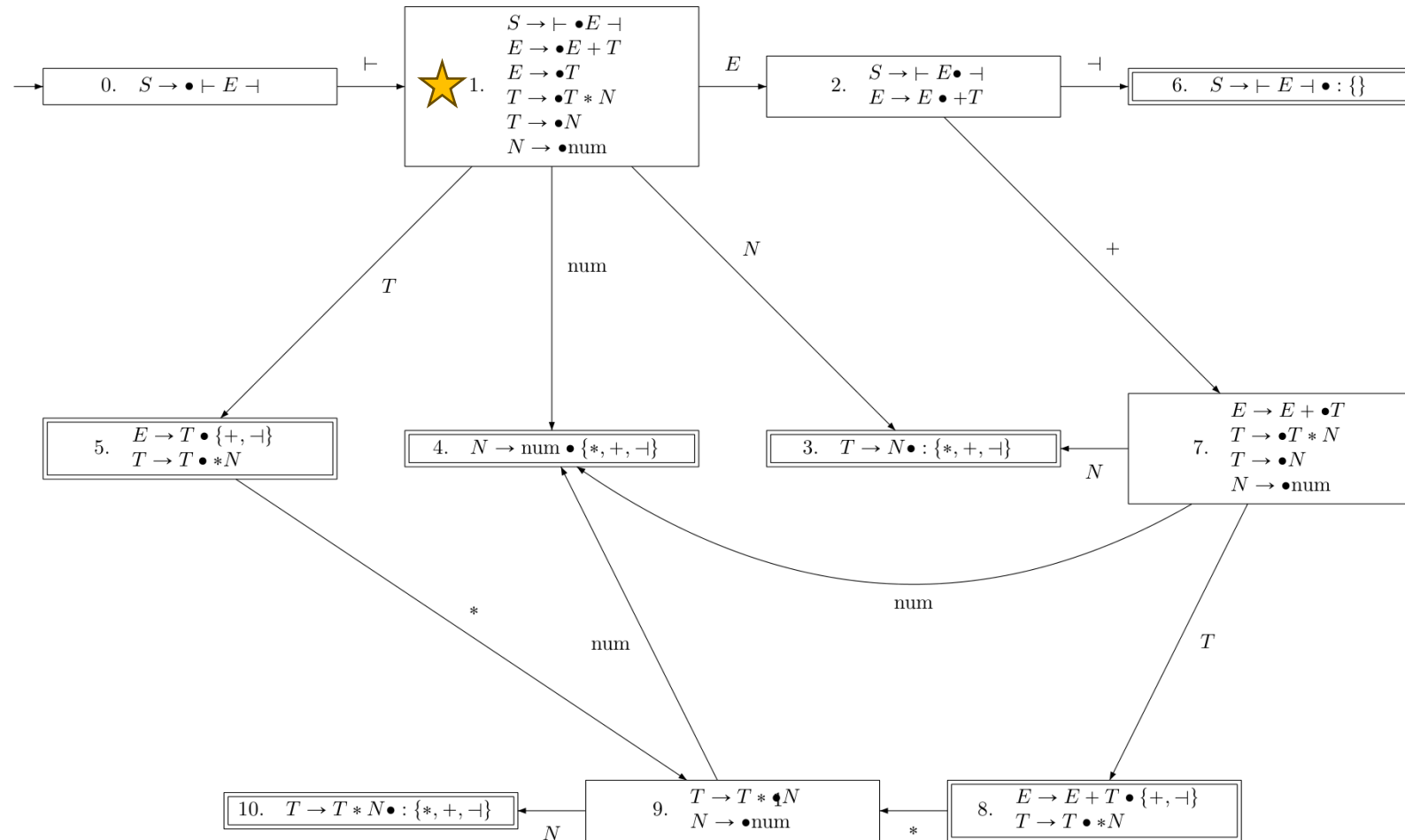
8. $E \to E + T \bullet \{+, \dashv\}$
$T \to T \bullet *N$

T

⊢

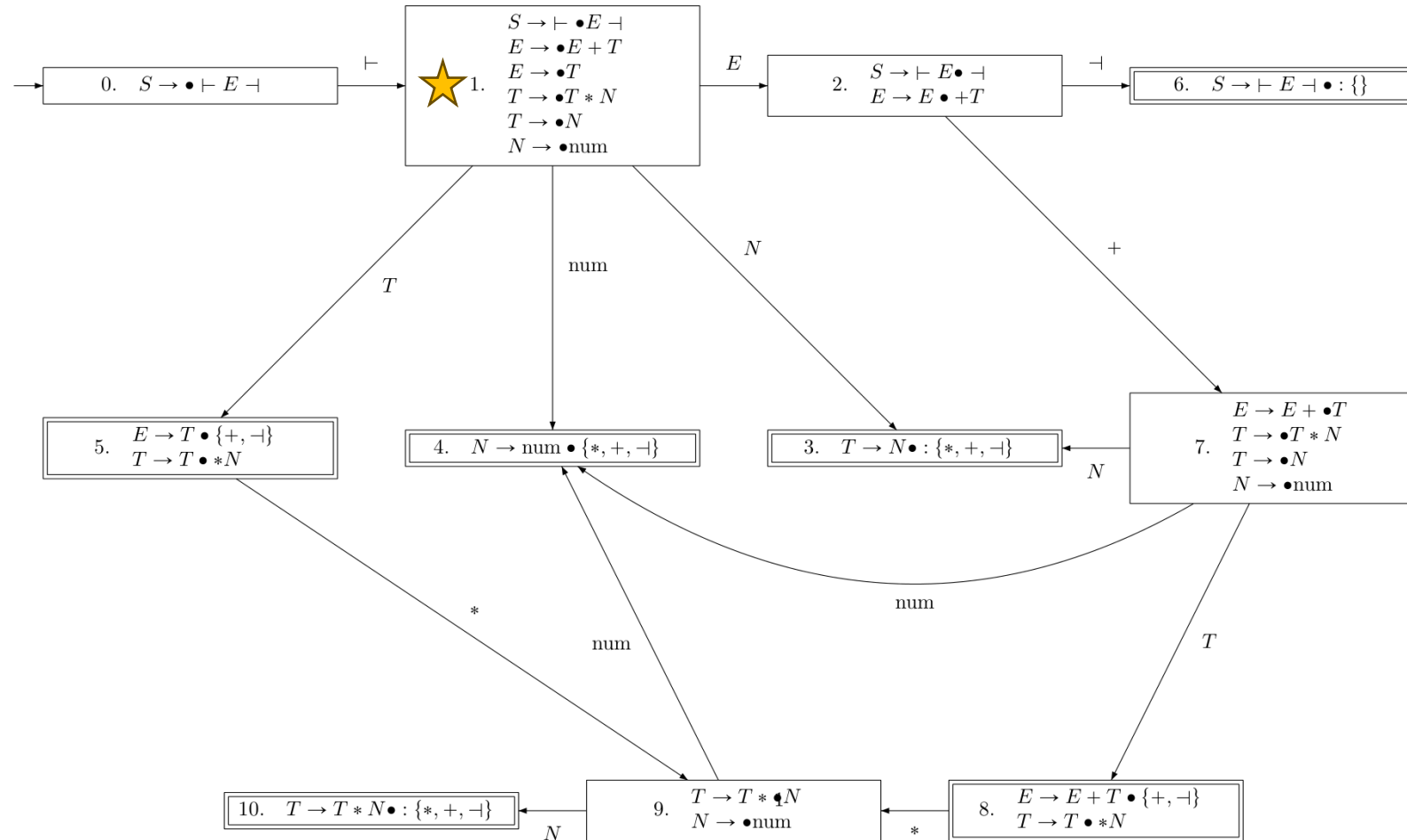Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1

Tree Stack Top

$$0. \quad S \to \bullet \vdash E \dashv$$

$$1. \quad
\begin{aligned}
S &\to \vdash \bullet E \dashv \\
E &\to \bullet E + T \\
E &\to \bullet T \\
T &\to \bullet T * N \\
T &\to \bullet N \\
N &\to \bullet \text{num}
\end{aligned}$$

$$2. \quad
\begin{aligned}
S &\to \vdash E \bullet \dashv \\
E &\to E \bullet + T
\end{aligned}$$

$$6. \quad S \to \vdash E \dashv \bullet : \{\}$$

$$5. \quad
\begin{aligned}
E &\to T \bullet \{+, \dashv\} \\
T &\to T \bullet *N
\end{aligned}$$

$$4. \quad N \to \text{num} \bullet \{*, +, \dashv\}$$

$$3. \quad T \to N \bullet : \{*, +, \dashv\}$$

$$7. \quad
\begin{aligned}
E &\to E + \bullet T \\
T &\to \bullet T * N \\
T &\to \bullet N \\
N &\to \bullet \text{num}
\end{aligned}$$

$$10. \quad T \to T * N \bullet : \{*, +, \dashv\}$$

$$9. \quad
\begin{aligned}
T &\to T * \bullet N \\
N &\to \bullet \text{num}
\end{aligned}$$

$$8. \quad
\begin{aligned}
E &\to E + T \bullet \{+, \dashv\} \\
T &\to T \bullet *N
\end{aligned}$$

Reducing by T → N:
Add N node as child
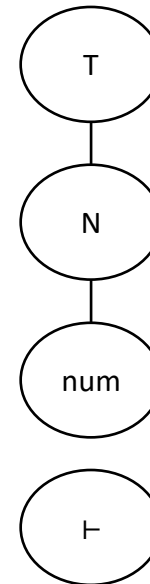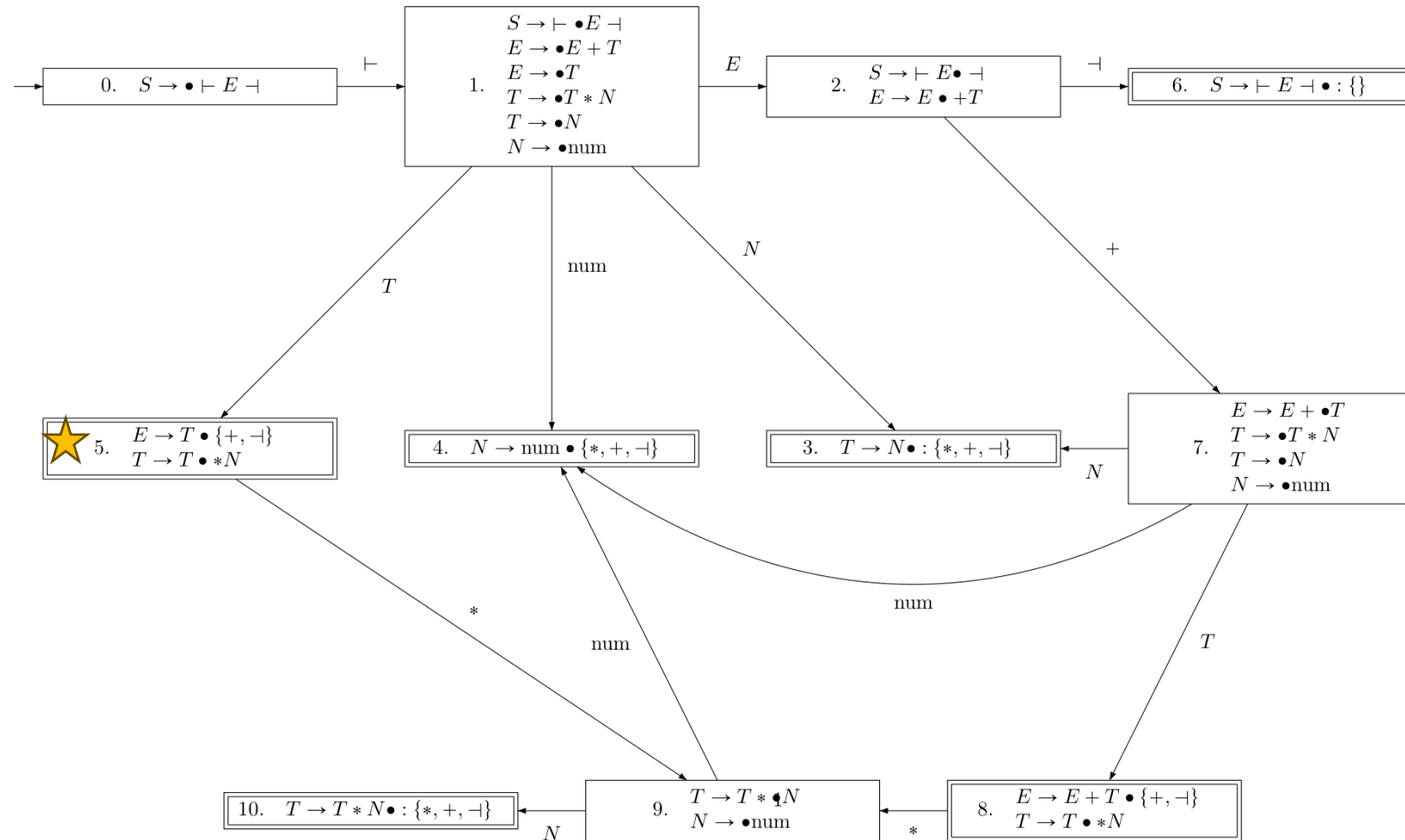
Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1 5

Tree Stack Top

Reducing by T → N:
Push T / 5

Tree Stack Bottom

Diagram contents:

0. $S \to \bullet \vdash E \dashv$

1. $S \to \vdash \bullet E \dashv$
   $E \to \bullet E + T$
   $E \to \bullet T$
   $T \to \bullet T * N$
   $T \to \bullet N$
   $N \to \bullet \text{num}$

2. $S \to \vdash E \bullet \dashv$
   $E \to E \bullet +T$

6. $S \to \vdash E \dashv \bullet : \{\}$

5. $E \to T \bullet \{+, \dashv\}$
   $T \to T \bullet *N$

4. $N \to \text{num} \bullet \{*, +, \dashv\}$

3. $T \to N \bullet : \{*, +, \dashv\}$

7. $E \to E + \bullet T$
   $T \to \bullet T * N$
   $T \to \bullet N$
   $N \to \bullet \text{num}$

10. $T \to T * N \bullet : \{*, +, \dashv\}$

9. $T \to T * \bullet N$
   $N \to \bullet \text{num}$

8. $E \to E + T \bullet \{+, \dashv\}$
   $T \to T \bullet *N$

Tree Stack (top to bottom): T, N, num, ⊢

⊢ num * num ⊣

State Stack: 0 1 5

Tree Stack Top

0. $S \rightarrow \bullet \vdash E \dashv$

⊢

1. $S \rightarrow \vdash \bullet E \dashv$
$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet$num

E

2. $S \rightarrow \vdash E \bullet \dashv$
$E \rightarrow E \bullet + T$

⊣

6. $S \rightarrow \vdash E \dashv \bullet : \{\}$

num

N

+

5. $E \rightarrow T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

4. $N \rightarrow$ num $\bullet \{*, +, \dashv\}$

3. $T \rightarrow N \bullet : \{*, +, \dashv\}$

7. $E \rightarrow E + \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet$num

N

*

num

num

T

10. $T \rightarrow T * N \bullet : \{*, +, \dashv\}$

N

9. $T \rightarrow T * \bullet N$
$N \rightarrow \bullet$num

*

8. $E \rightarrow E + T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

T

N

num

⊢

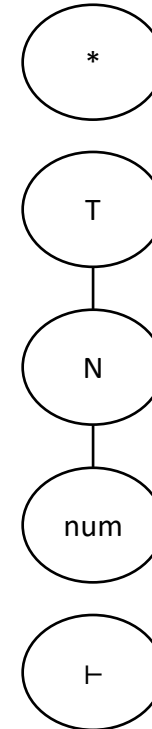Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1 5 9

Tree Stack Top

$0. \quad S \to \bullet \vdash E \dashv$

$1.$
$S \to \vdash \bullet E \dashv$
$E \to \bullet E + T$
$E \to \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet \text{num}$

$2. \quad S \to \vdash E \bullet \dashv$
$E \to E \bullet + T$

$6. \quad S \to \vdash E \dashv \bullet : \{\}$

$5.$
$E \to T \bullet \{+, \dashv\}$
$T \to T \bullet * N$

$4. \quad N \to \text{num} \bullet \{*, +, \dashv\}$

$3. \quad T \to N \bullet : \{*, +, \dashv\}$

$7.$
$E \to E + \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet \text{num}$

$10. \quad T \to T * N \bullet : \{*, +, \dashv\}$

$9.$
$T \to T * \bullet N$
$N \to \bullet \text{num}$

$8.$
$E \to E + T \bullet \{+, \dashv\}$
$T \to T \bullet * N$

Edges labeled: ⊢, E, ⊣, T, num, N, +, N, *, num, num, T, N, *
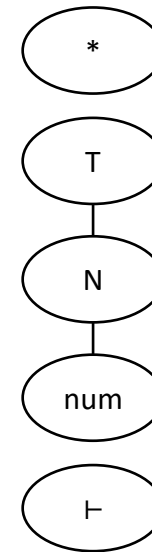
Tree Stack (top to bottom): *, T, N, num, ⊢
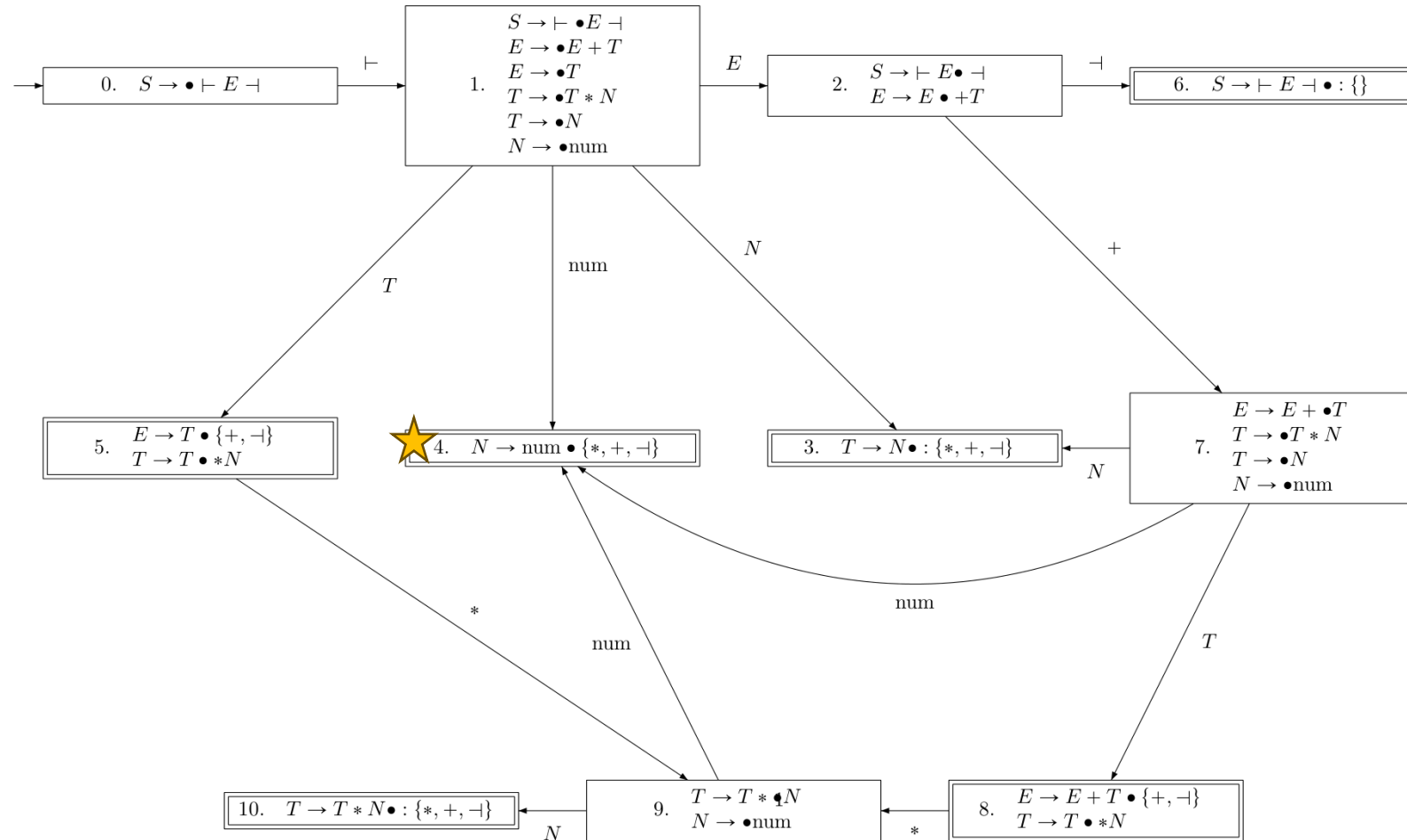
Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1 5 9

Tree Stack Top

(No change to stack contents, just squishing it to fit more trees on the slide)

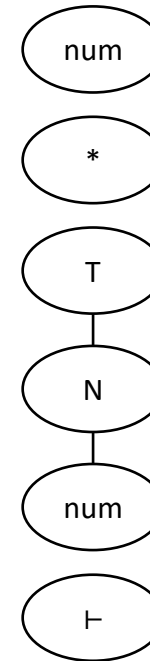Tree Stack Bottom
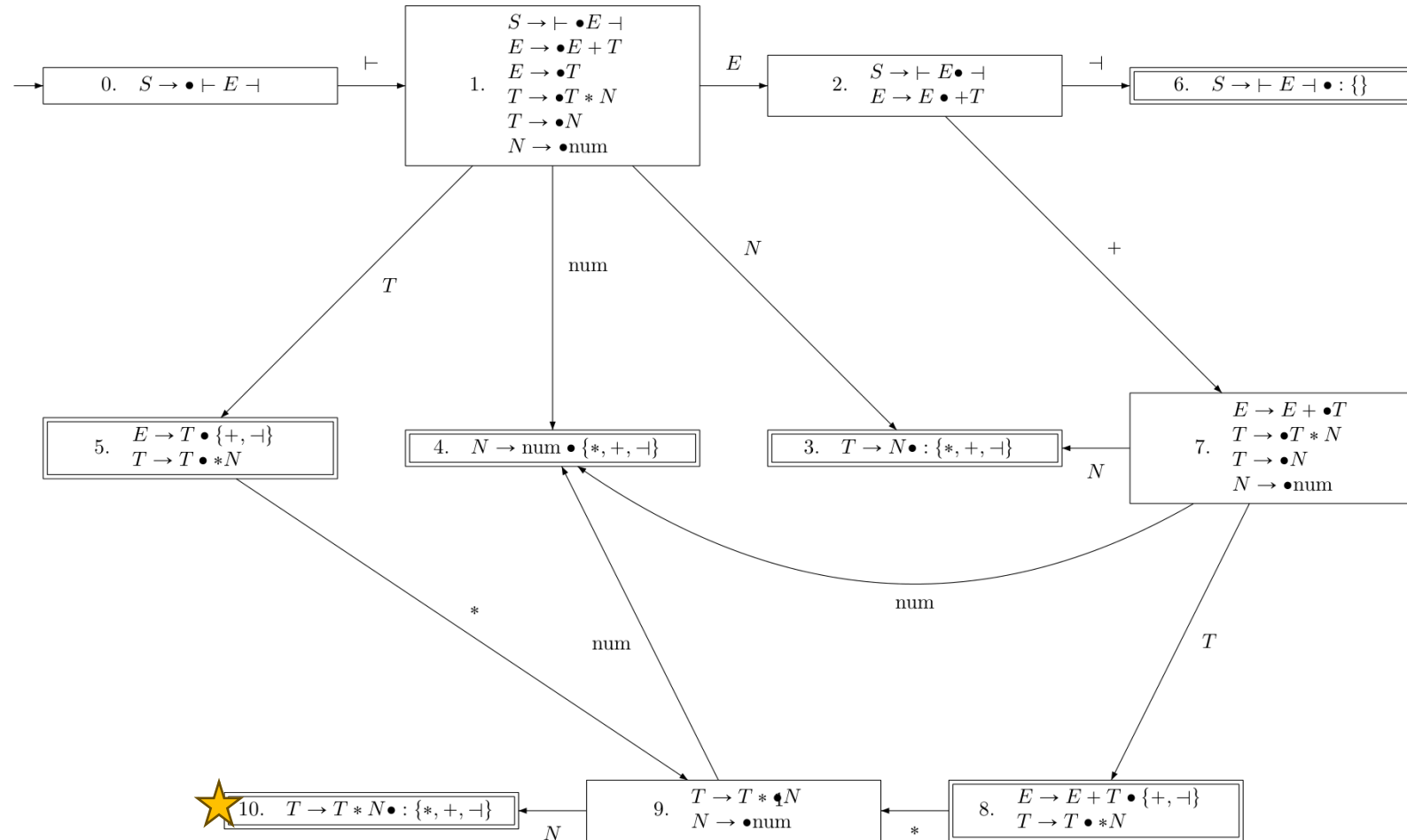
⊢ num * num ⊣

State Stack: 0 1 5 9 4

Tree Stack Top

→ | 0. $S \to \bullet \vdash E \dashv$ |

⊢ →

| 1. | $S \to \vdash \bullet E \dashv$
 $E \to \bullet E + T$
 $E \to \bullet T$
 $T \to \bullet T * N$
 $T \to \bullet N$
 $N \to \bullet \text{num}$ |

E →

| 2. | $S \to \vdash E \bullet \dashv$
 $E \to E \bullet + T$ |

⊣ →

| 6. | $S \to \vdash E \dashv \bullet : \{\}$ |

T ↓   num ↓   N ↓   + ↓

| 5. | $E \to T \bullet \{+, \dashv\}$
 $T \to T \bullet * N$ |

⭐ | 4. | $N \to \text{num} \bullet \{*, +, \dashv\}$ |

| 3. | $T \to N \bullet : \{*, +, \dashv\}$ |

| 7. | $E \to E + \bullet T$
 $T \to \bullet T * N$
 $T \to \bullet N$
 $N \to \bullet \text{num}$ |

*   num   num   T

| 10. | $T \to T * N \bullet : \{*, +, \dashv\}$ |

N ←

| 9. | $T \to T * \bullet N$
 $N \to \bullet \text{num}$ |

* ←

| 8. | $E \to E + T \bullet \{+, \dashv\}$
 $T \to T \bullet * N$ |

num

*

T

N

num

⊢

Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1 5 9 10

Tree Stack Top

0. $S \rightarrow \bullet \vdash E \dashv$

1.
$S \rightarrow \vdash \bullet E \dashv$
$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet \text{num}$

2.
$S \rightarrow \vdash E \bullet \dashv$
$E \rightarrow E \bullet + T$

6. $S \rightarrow \vdash E \dashv \bullet : \{\}$

5.
$E \rightarrow T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

4. $N \rightarrow \text{num} \bullet \{*, +, \dashv\}$

3. $T \rightarrow N \bullet : \{*, +, \dashv\}$

7.
$E \rightarrow E + \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet \text{num}$

10. $T \rightarrow T * N \bullet : \{*, +, \dashv\}$

9.
$T \rightarrow T * \bullet N$
$N \rightarrow \bullet \text{num}$

8.
$E \rightarrow E + T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

N

num

*

T

N

num

⊢

Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1 5 9

Tree Stack Top

Reducing by T → T * N:
Pop N / 10

N
num

$*$
T
N
num
$⊢$

Tree Stack Bottom

0. $S \rightarrow \bullet \vdash E \dashv$

1. $S \rightarrow \vdash \bullet E \dashv$
$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet num$

2. $S \rightarrow \vdash E \bullet \dashv$
$E \rightarrow E \bullet + T$

6. $S \rightarrow \vdash E \dashv \bullet : \{\}$

5. $E \rightarrow T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

4. $N \rightarrow num \bullet \{*, +, \dashv\}$

3. $T \rightarrow N \bullet : \{*, +, \dashv\}$

7. $E \rightarrow E + \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet num$

10. $T \rightarrow T * N \bullet : \{*, +, \dashv\}$

9. $T \rightarrow T * \bullet N$
$N \rightarrow \bullet num$

8. $E \rightarrow E + T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

⊢ num * num ⊣

State Stack: 0 1 5

Tree Stack Top

Reducing by T → T * N:
Pop * / 9

N

num

*

T

N

num

⊢

Tree Stack Bottom

0. $S \rightarrow \bullet \vdash E \dashv$

⊢

1.
$S \rightarrow \vdash \bullet E \dashv$
$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet \text{num}$

E

2.
$S \rightarrow \vdash E \bullet \dashv$
$E \rightarrow E \bullet + T$

⊣

6. $S \rightarrow \vdash E \dashv \bullet : \{\}$

T

num

N

+

5.
$E \rightarrow T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

4. $N \rightarrow \text{num} \bullet \{*, +, \dashv\}$

3. $T \rightarrow N \bullet : \{*, +, \dashv\}$

7.
$E \rightarrow E + \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet \text{num}$

N

num

*

num

T

10. $T \rightarrow T * N \bullet : \{*, +, \dashv\}$

N

9.
$T \rightarrow T * \bullet N$
$N \rightarrow \bullet \text{num}$

*

8.
$E \rightarrow E + T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

⊢ num * num ⊣

State Stack: 0 1

Tree Stack Top

Reducing by T → T * N:
Pop T / 5

0. $S \rightarrow \bullet \vdash E \dashv$

⊢

1. $S \rightarrow \vdash \bullet E \dashv$
$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet \text{num}$

E

2. $S \rightarrow \vdash E \bullet \dashv$
$E \rightarrow E \bullet + T$

⊣

6. $S \rightarrow \vdash E \dashv \bullet : \{\}$

5. $E \rightarrow T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

4. $N \rightarrow \text{num} \bullet \{*, +, \dashv\}$

3. $T \rightarrow N \bullet : \{*, +, \dashv\}$

7. $E \rightarrow E + \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet \text{num}$

10. $T \rightarrow T * N \bullet : \{*, +, \dashv\}$

9. $T \rightarrow T * \bullet N$
$N \rightarrow \bullet \text{num}$

8. $E \rightarrow E + T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

Tree Stack Bottom

⊢ num * num ⊣

Tree Stack Top

Reducing by T → T * N:
Create node for T

Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1

Tree Stack Top

Reducing by T → T * N:
Add T * N as children

Tree Stack Bottom

→ 0. $S \to \bullet \vdash E \dashv$

⊢

1. ★
$S \to \vdash \bullet E \dashv$
$E \to \bullet E + T$
$E \to \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet \text{num}$

E

2. $S \to \vdash E \bullet \dashv$
$E \to E \bullet + T$

⊣

6. $S \to \vdash E \dashv \bullet : \{\}$

5. $E \to T \bullet \{+, \dashv\}$
$T \to T \bullet *N$

4. $N \to \text{num} \bullet \{*, +, \dashv\}$

3. $T \to N \bullet : \{*, +, \dashv\}$

7.
$E \to E + \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet \text{num}$

10. $T \to T * N \bullet : \{*, +, \dashv\}$

9.
$T \to T * \bullet N$
$N \to \bullet \text{num}$

8. $E \to E + T \bullet \{+, \dashv\}$
$T \to T \bullet *N$

T

num

N

+

N

*

num

num

N

*

T

T

⊢

⊢ num * num ⊣

State Stack: 0 1 5

Tree Stack Top

Reducing by T → T * N:
Push T / 5

$0. \quad S \to \bullet \vdash E \dashv$

$1. \quad S \to \vdash \bullet E \dashv$
$E \to \bullet E + T$
$E \to \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet num$

$2. \quad S \to \vdash E \bullet \dashv$
$E \to E \bullet + T$

$6. \quad S \to \vdash E \dashv \bullet : \{\}$

$5. \quad E \to T \bullet \{+, \dashv\}$
$T \to T \bullet * N$

$4. \quad N \to num \bullet \{*, +, \dashv\}$

$3. \quad T \to N \bullet : \{*, +, \dashv\}$

$7. \quad E \to E + \bullet T$
$T \to \bullet T * N$
$T \to \bullet N$
$N \to \bullet num$

$10. \quad T \to T * N \bullet : \{*, +, \dashv\}$

$9. \quad T \to T * \bullet N$
$N \to \bullet num$

$8. \quad E \to E + T \bullet \{+, \dashv\}$
$T \to T \bullet * N$

Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1 5

Tree Stack Top

$$0. \quad S \to \bullet \vdash E \dashv$$

$$1. \quad \begin{aligned} S &\to \vdash \bullet E \dashv \\ E &\to \bullet E + T \\ E &\to \bullet T \\ T &\to \bullet T * N \\ T &\to \bullet N \\ N &\to \bullet \text{num} \end{aligned}$$

$$2. \quad \begin{aligned} S &\to \vdash E \bullet \dashv \\ E &\to E \bullet + T \end{aligned}$$

$$6. \quad S \to \vdash E \dashv \bullet : \{\}$$

$$5. \quad \begin{aligned} E &\to T \bullet \{+, \dashv\} \\ T &\to T \bullet * N \end{aligned}$$

$$4. \quad N \to \text{num} \bullet \{*, +, \dashv\}$$

$$3. \quad T \to N \bullet : \{*, +, \dashv\}$$

$$7. \quad \begin{aligned} E &\to E + \bullet T \\ T &\to \bullet T * N \\ T &\to \bullet N \\ N &\to \bullet \text{num} \end{aligned}$$

$$10. \quad T \to T * N \bullet : \{*, +, \dashv\}$$

$$9. \quad \begin{aligned} T &\to T * \bullet N \\ N &\to \bullet \text{num} \end{aligned}$$

$$8. \quad \begin{aligned} E &\to E + T \bullet \{+, \dashv\} \\ T &\to T \bullet * N \end{aligned}$$

Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1 2

Tree Stack Top

0. $S \rightarrow \bullet \vdash E \dashv$

⊢

1. $S \rightarrow \vdash \bullet E \dashv$
$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet \text{num}$

E

⭐ 2. $S \rightarrow \vdash E \bullet \dashv$
$E \rightarrow E \bullet + T$

⊣

6. $S \rightarrow \vdash E \dashv \bullet : \{\}$

5. $E \rightarrow T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

4. $N \rightarrow \text{num} \bullet \{*, +, \dashv\}$

3. $T \rightarrow N \bullet : \{*, +, \dashv\}$

7. $E \rightarrow E + \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet \text{num}$

10. $T \rightarrow T * N \bullet : \{*, +, \dashv\}$

9. $T \rightarrow T * \bullet N$
$N \rightarrow \bullet \text{num}$

8. $E \rightarrow E + T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

Tree Stack Bottom

⊢ num * num ⊣

State Stack: 0 1 2 6

Tree Stack Top

$0.\quad S \rightarrow \bullet \vdash E \dashv$

$\vdash$

$1.\quad$
$S \rightarrow \vdash \bullet E \dashv$
$E \rightarrow \bullet E + T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet \text{num}$

E

$2.\quad$
$S \rightarrow \vdash E \bullet \dashv$
$E \rightarrow E \bullet + T$

$\dashv$

⭐ $6.\quad S \rightarrow \vdash E \dashv \bullet : \{\}$

T

num

N

$+$

$5.\quad$
$E \rightarrow T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

$4.\quad N \rightarrow \text{num} \bullet \{*, +, \dashv\}$

$3.\quad T \rightarrow N \bullet : \{*, +, \dashv\}$

$7.\quad$
$E \rightarrow E + \bullet T$
$T \rightarrow \bullet T * N$
$T \rightarrow \bullet N$
$N \rightarrow \bullet \text{num}$

N

num

$*$

num

T

$10.\quad T \rightarrow T * N \bullet : \{*, +, \dashv\}$

N

$9.\quad$
$T \rightarrow T * \bullet N$
$N \rightarrow \bullet \text{num}$

$*$

$8.\quad$
$E \rightarrow E + T \bullet \{+, \dashv\}$
$T \rightarrow T \bullet * N$

Tree Stack Bottom

⊢ num * num ⊣

LR automaton / parse states:

0. $S \to \bullet \vdash E \dashv$

1. $S \to \vdash \bullet E \dashv$
   $E \to \bullet E + T$
   $E \to \bullet T$
   $T \to \bullet T * N$
   $T \to \bullet N$
   $N \to \bullet \text{num}$

2. $S \to \vdash E \bullet \dashv$
   $E \to E \bullet + T$

3. $T \to N \bullet : \{*, +, \dashv\}$

4. $N \to \text{num} \bullet \{*, +, \dashv\}$

5. $E \to T \bullet \{+, \dashv\}$
   $T \to T \bullet * N$

6. $S \to \vdash E \dashv \bullet : \{\}$

7. $E \to E + \bullet T$
   $T \to \bullet T * N$
   $T \to \bullet N$
   $N \to \bullet \text{num}$

8. $E \to E + T \bullet \{+, \dashv\}$
   $T \to T \bullet * N$

9. $T \to T * \bullet N$
   $N \to \bullet \text{num}$

10. $T \to T * N \bullet : \{*, +, \dashv\}$

Semantic Analysis

Also known as Context-Sensitive Analysis

The Stages of Compilation

- The compilation process can be broadly divided into four stages.
    - **Scanning:** Group the individual characters in the source into meaningful chunks called tokens, and detect errors related to syntax of tokens.
    - **Parsing:** Group the tokens into meaningful high-level structures like statements and expressions, and detect errors related to syntax of structures.
    - **Semantic Analysis:** Gather further information about the semantics (meaning) of the program, e.g. scope of identifiers and types of expressions, and detect errors related to semantics.
        - The program should be free of compile-time errors after this stage.
    - **Code Generation:** Translate each structural component of the program into the target language using the information obtained in the previous stages.

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        - The program should be free of compile-time errors after this stage.
    - **Code Generation:** Translate each structural component of the program into the target language using the information obtained in the previous stages.

The WLP4 Programming Language

- WLP4 (Waterloo Language Plus Pointers Plus Procedures) is the programming language we are writing a compiler for in this course.
- It is a (very small) subset of C++ that includes the following:
  - Variables of int (32-bit signed integer) or int* (pointer to int) type
  - Arithmetic expressions with brackets and the operations: + - * / %
  - Printing the value of an int variable
  - If/else statements and while loops, with conditions using the comparison operators: == != < > <= >=
  - Null pointers, pointer operations (dereference/address-of), pointer arithmetic
  - Dynamic memory allocation for int arrays (new/delete)
  - Procedures that take any amount/type of arguments and return an int value (and a special "wain" procedure which works like the C/C++ "main" function)

Semantic Errors in WLP4

- The semantic errors one needs to check for depend on the language.
- Many errors broadly fall into one of two categories.
- **Name errors** are errors related to identifiers and their meanings.
  - A name is used but a definition of the name cannot be found.
  - A name is defined multiple times and there is no way to disambiguate.
- **Type errors** are errors related to the types of expressions.
  - Adding two integers is valid, but adding two pointers is invalid.
  - Calling "delete" on an expression that is not a pointer is invalid.
  - If a procedure expects an integer parameter, passing a pointer is invalid.

Detecting Semantic Errors

- To parse programming languages, we had to move from regular languages to the wider class of context-free languages.

- Technically, there is a class called **context-sensitive languages** that we could use to describe semantically correct programs.

- Semantic analysis is sometimes called **context-sensitive analysis**.

- However, writing context-sensitive grammars and context-sensitive parsers is difficult and nobody does it.

- It is much easier to just analyze the **parse tree** obtained from the parsing phase than to approach this in a language-theoretic way.

Working with Parse Trees

- You can tell what kind of feature or aspect of the program you are looking at by examining the rule that defines the parse tree node.

- For example, the rule for the main (wain) function looks like:

main → INT WAIN LPAREN dcl COMMA dcl RPAREN LBRACE dcls statements RETURN expr SEMI RBRACE

- The rule for a while loop looks like:

statement → WHILE LPAREN test RPAREN LBRACE statements RBRACE

- When drawing parse trees, we usually just draw one symbol (terminal or nonterminal) in each node.

- Project 3 asks you to store the corresponding **CFG rule** in each parse tree node that corresponds to a nonterminal.