CS 245 - Lecture 01 - Introduction to CS 245

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Outline

1. Introduction to CS 245
2. Introduction to Propositional Logic
   1. Applications of Logic in Computer Science
   2. Why We Need Formal Languages
3. Propositions
4. Connectives
5. Translations Between English and Propositional Logic
6. Logical Arguments
1. Will you lecture from slides, or on the blackboard / document camera?

A: I intend to lecture using prepared slides. Whenever I need to go off-script, I will write on the document camera, and add those notes to the prepared slides afterward. My Lecture Notes (on which my prepared slides will be based) are posted on LEARN.

2. How will tutorials work?

A: Instructors will prepare material for the IAs to present during the hour. These materials will be posted on LEARN, at the end of the day on Friday.
3. Will all presentation / evaluation be “writable by hand”?
   
   **A:** Yes. CS 245 is very much like a MATH course. We will not code; instead we will work with logical objects that will help us to **think about coding**.

4. Do most students pass CS 245?
   
   **A:** Yes! The instructors are not seeking to “weed out” anyone. However we must evaluate based on our course syllabus.

5. When will Crowdmark Assignments and Marked Quizzes be due?

   **A:** 11:59 PM.
6. **Will any Marked Quizzes be in-person?**

**A:** No. All Marked Quizzes will be delivered on LEARN.

7. **What is Crowdmark?**

**A:** Crowdmark is a website, on which:

1. instructors create / distribute assignments,
2. students submit their answers to assignment questions (.pdf format is preferred; graphic formats are also allowed),
3. TAs grade the student answers, and
4. instructors release marked assignments back to the students.
8 Is the historical material in Logic01 examinable?

A: No. The history is for interest. The **translation** material from Logic01 will appear on assignments and exams.

9 Can we use the smartphone version of the iClicker software?

A: Yes!
1 Q: How will the course run?

A: Refer to the course website for high-level answers:
https://student.cs.uwaterloo.ca/~cs245

2 Q: How will iClickers work?

A: Bring your iClicker to class. Use your iClicker to answer occasional questions which will be presented during the lectures. There are no marks attached to iClicker use - it is 100% optional.

3 Q: What homework do I have before the next lecture?

A: Read the Logic01 and Logic02 slide decks, and come to the next lecture prepared to ask any questions that you have about them.
Setting Course Expectations

1. You should not expect that CS 245 will (directly) improve your coding skills.

2. You should instead expect that CS 245 will make you a more effective thinker about coding.

3. This will ultimately improve your coding, once you have assimilated CS 245 into your thinking.
Topics in CS For Which Logic Is Relevant

1. SAT-solvers (Propositional Logic)
2. Database Analysis (First-Order Logic)
3. Properties of the Natural Numbers - Peano Axioms
4. Program Verification
5. Decidability and Undecidability
6. Definability and Undefinability
7. Provability and Unprovability
8. Artificial Intelligence
9. and many more ...
Motivation: We are often tempted to rely on our own understanding and intuition exclusively. However sometimes an apparently reasonable decision problem (Definition 1) has subtleties.

Definition 1

A decision problem is a problem which calls for an answer of either yes (1) or no (0), given some input.
Definition 2

A **paradox** is a declarative statement that

1. cannot be true, and
2. cannot be false.

Examples:

1. “This sentence is false.”
The Barber Paradox (Temporarily assume a universe of only men.)

There is a barber who is said to shave each man, if and only if that man does not shave himself. Q: Does the barber shave himself?

- Then if the barber shaves himself, then it is because he does not shave himself, and in turn this is because he does shave himself.

- Also, if the barber does not shaves himself, then it is because he shaves himself.

- “Does the barber shave himself?” is unanswerable.
### Barey’s Paradox

Consider the set of natural numbers \( \mathbb{N} = \{0, 1, 2, 3, ...\} \).

**Remarks:**

1. Unlike what you were likely told in MATH 135, in CS 245 we take 0 to be the first Natural number.
Definition 3

We say that a Natural Number, \( n \), has a **compact definition** if there is an English sentence of at most 200 characters that uniquely defines the number \( n \).
Examples:

1. “\( n \) is 3.”
2. “\( n \) is the difference of 10 and 7.”
3. “\( n \) is one million.”
4. “\( n \) is one million to the power of one million.”
5. “\( n \) is the number of cells in my body.”
6. “\( n \) is the number of grains of sand on a California beach.”
7. The sentence “\( n \) is even.” identifies no Natural number.
8. The sentence “Fruit flies like a banana.” identifies no Natural number.
Let $B$ be the set of all Natural numbers that have a compact definition.

**Q:** Is $B$ a finite set? (CQ 2)
Let $B$ be the set of all Natural numbers that have a compact definition.

**Q:** Is $B$ a finite set? (CQ 2)

**A:** Yes: $|B| \leq 40^{200} < \infty$ (where we get 40 from 26 letters, 10 digits and a few punctuation marks). There are only finitely many compact English descriptions.

Since $B$ is finite and $\mathbb{N}$ is infinite, we may consider the first Natural number, $x$, which does not have a compact definition.
Q: Is $x \in B$?

A:

- If $x \in B$, then we have a contradiction, since by construction $x$ is the first Natural Number such that $x \notin B$.

- If $x \notin B$, then by construction there exists a way to define $x$ by an English sentence of length $\leq 200$ (the preceding description of $x$ constitutes a compact definition). So $x \in B$.

Then this shows the paradox.
Morals:

1. The presence of such paradoxes, arising from descriptions in some natural language (English in these cases), tells us that we will need formal languages to study logic carefully.

2. We will see that paradoxes cannot occur in formal logic: every statement (formula) in formal logic is either true (1) or false (0) (and not both) in some context.
Definition 4

A proposition is a declarative sentence that is either true or false, in some context.

Examples:

1. If I feed my fish, and I change my fish’s tank filter, then my fish will be healthy. (compound, not simple)
Definition 5

An **atomic (simple) proposition** is a proposition that cannot be broken down into smaller propositions. A proposition that is not atomic (simple) is called **compound**.

In particular, the presence of any **connective** indicates that the proposition is compound.

**Examples:**

1. My fish will be healthy. (atomic a.k.a. simple)

**Remarks:**

1. We assign **proposition symbols** like p, q and r to represent simple propositions when we translate from English into Propositional Logic.
You likely met these connectives and their truth tables in MATH 135.

**Unary**

1. $\neg$ (negation)

**Binary**

1. $\land$ (conjunction)
2. $\lor$ (disjunction)
3. $\rightarrow$ (implication)

**Truth Table:**

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$(p \rightarrow q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
<td>1</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

4. $\leftrightarrow$ (equivalence)

**Truth Table:**

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$(p \leftrightarrow q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0</td>
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<td>1</td>
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</tbody>
</table>
First Example, As We Have Limited Time Remaining: the compound proposition from above.

1. If I feed my fish, and I change my fish’s tank filter, then my fish will be healthy.

\[ ((p \land q) \rightarrow r) \], where

- p: I feed my fish.
- q: I change my fish’s tank filter.
- r: My fish will be healthy.
Remarks:

1. Choose your atomic propositions (which will correspond to proposition symbols) to be positive statements (i.e. without any embedded negations). E.g. translate “I did not change my fish’s tank filter” as ($\neg q$), where $q$ means “I changed my fish’s tank filter”.

2. Negated statements are compound, not atomic.

3. Assemble atomic propositions, using the appropriate connectives to express the provided English sentence.

4. Implications are particularly interesting here.

5. You will get some more practice at this on next week’s tutorial, and on the first marked quiz and the first assignment.
Translate the following examples from English into Formulas of Propositional Logic:

1. She is clever and hard working. 
   \((p \wedge q)\), where
   - \(p\): She is clever.
   - \(q\): She is hard working.

2. He is clever but not hard working. 
   \((p \wedge (\neg q))\), where
   - \(p\): He is clever.
   - \(q\): He is hard working.
If it rains, then he will be at home; otherwise he will go to the market or he will go to school.

\[((p \rightarrow q) \land ((\neg p) \rightarrow (r \lor s)))\], where

- \(p\): It rains.
- \(q\): He will be at home.
- \(r\): He will go to the market.
- \(s\): He will go to school.

Note that we need \(\land\) and not \(\lor\) as the last binary connective in this formula!
The sum of two integers is even if and only if both integers are even or both integers are odd.

\[(p \leftrightarrow ((q_1 \land q_2) \lor (r_1 \land r_2))),\]

where

- \(p\): The sum of the two integers is even.
- \(q_1\): The first integer is even.
- \(q_2\): The second integer is even.
- \(r_1\): The first integer is odd.
- \(r_2\): The second integer is odd.
Remarks:

1. We could replace $r_1$ by $\neg q_1$, and $r_2$ by $\neg q_2$, but this would take us further away from the original English statement.
Translate the following examples from Formulas of Propositional Logic into English: Use the atoms:

- $p$: Today is Sunday.
- $q$: I do homework.
- $r$: I watch TV.
(q ↔ (¬p))

“I do homework if and only if today is not Sunday.”, or

“I do homework every day except Sunday.” (less direct, but a bit more natural).

(q ∨ r)

“I do homework or I watch TV.”

“I do homework unless I watch TV.” (N.B. “Unless” is messy in English - sometimes it indicates an inclusive or the way we have translated it here, and other times it indicates an exclusive or)
(p → r)

- “On Sundays I watch TV.”
- “If today is Sunday then I watch TV.”, or
- “I watch TV if today is Sunday.”, or
- “Today is Sunday only if I watch TV.”
Given the premises:

1. If I study before my mid-term, and I get 8 hours’ sleep before my mid-term, then I will pass my mid-term.
2. I studied before my mid-term.
3. I did not pass my mid-term.

We may conclude:

1. Therefore I must not have gotten 8 hours’ sleep before my mid-term.

Q: Is this argument “convincing” to you?

A: I find this argument “convincing”. I will justify this assertion, soon.

**Shorthand Used In Slides:** Separate the premises and the conclusion using a horizontal line.