University of Waterloo
CS 341 Fall 2022
Midterm Exam Reference Sheet

Tuesday November 1st, 2022 at 7:00pm - 8:50pm

Master Theorem: Given

$$T(n) = aT(\frac{n}{b}) + cn^k$$

where $a \geq 1, b > 1, c > 0, k \geq 0$, then

$$T(n) \in \begin{cases} \Theta(n^k) & \text{if } a < b^k \text{ i.e. } \log_b a < k \\ \Theta(n^k \log n) & \text{if } a = b^k \\ \Theta(n^{\log_b a}) & \text{if } a > b^k \end{cases}$$

Arithmetic sequence:

$\sum_{i=0}^{n-1} i = ???$ $\qquad$ $\sum_{i=0}^{n-1}(a+di) = na + \frac{dn(n-1)}{2} \in \Theta(n^2)$ if $d \neq 0$.

Geometric sequence:

$\sum_{i=0}^{n-1} 2^i = ???$ $\qquad$ $\sum_{i=0}^{n-1} a\, r^i = \begin{cases} a\frac{r^n-1}{r-1} & \in \Theta(r^{n-1}) \quad \text{if } r > 1 \\ na & \in \Theta(n) \quad \text{if } r = 1 \\ a\frac{1-r^n}{1-r} & \in \Theta(1) \quad \text{if } 0 < r < 1. \end{cases}$

Harmonic sequence:

$\sum_{i=1}^{n} \frac{1}{i} = ???$ $\qquad$ $H_n := \sum_{i=1}^{n} \frac{1}{i} = \ln n + \gamma + o(1) \in \Theta(\log n)$

A few more:

$\sum_{i=1}^{n} \frac{1}{i^2} = ???$ $\qquad$ $\sum_{i=1}^{n} \frac{1}{i^2} = \frac{\pi^2}{6} \in \Theta(1)$

$\sum_{i=1}^{n} i^k = ???$ $\qquad$ $\sum_{i=1}^{n} i^k \in \Theta(n^{k+1})$ for $k \geq 0$

Logarithms:

- $c = \log_b(a)$ means $b^c = a$. E.g. $n = 2^{\log n}$.
- $\log(a)$ (in this course) means $\log_2(a)$
- $\log(a \cdot c) = \log(a) + \log(c)$, $\log(a^c) = c \log(a)$
- $\log_b(a) = \frac{\log_c a}{\log_c b} = \frac{1}{\log_a(b)}$, $a^{\log_b c} = c^{\log_b a}$
- $\ln(x) = \text{natural log} = \log_e(x)$, $\frac{d}{dx} \ln x = \frac{1}{x}$
- concavity: $\alpha \log x + (1-\alpha) \log y \leq \log(\alpha x + (1-\alpha)y)$ for $0 \leq \alpha \leq 1$

Factorial:

- $n! := n(n-1)(n-2) \cdots 2 \cdot 1 =$ # ways to permute n elements
- $\log(n!) = \log n + \log(n-1) + \cdots + \log 2 + \log 1 \in \Theta(n \log n)$