ASSIGNMENT 3

DUE: Tuesday October 5, 11:59 PM. DO NOT COPY. ACKNOWLEDGE YOUR SOURCES.

Please read http://www.student.cs.uwaterloo.ca/~cs341 for general instructions and policies.

Note: “Giving” an algorithm means doing the four parts (i)–(iv) as described on the course web page. This goes without saying for all future assignments and exams.

1. [10 marks] You’ve been hired in an IT help desk. When you start the day, you are given a set of \( n \) queries from people. You can handle them in any order. The length of time to answer the \( i \)th query is \( \ell_i \) minutes (you are given these values ahead of time). Let \( L = \sum_{i=1}^{n} \ell_i \). If you complete query \( i \) at time \( t_i \), you get a bonus of \( L - t_i \) dollars (because the person is so happy they didn’t have to wait until the final time \( L \)).

Give an algorithm to find the ordering of queries that maximizes your bonus.

2. [2 marks for each one that does not work; 5 for each one that does]

You have now been promoted to manager because of so many bonuses. There are \( k \) people working under you. Each one can work for \( H \) minutes in the day. You have \( n \) queries, the \( i \)th query takes \( \ell_i \) minutes. You want to assign the queries to the workers so that each person has \( \leq H \) minutes of work. (If there are any queries left over, you have to stay late and do them yourself.)

For each of the following greedy strategies, prove whether it works or not.

(a) Assign the queries one by one from their input order, giving the query to the first worker who can fit it into their workday.

(b) Sort the queries by \( \ell_i \) from min to max and then do (a).

(c) Sort the queries by \( \ell_i \) from max to min and then do (a).

3. [10 marks] Your next job at the IT company is to assign people to the emergency call-in desk so that there is at least one person on call at all times from the start time \( S \) to the end time \( T \). There are \( n \) people, and the \( i \)th one is available from start time \( s_i \) to end time \( t_i \). (They hold weird hours and no one is willing do just part of their shift. It is ok to have multiple people on call at the same time.) Suppose the input is already sorted by start time so \( S \leq s_1 \leq s_2 \cdots \leq s_n \leq T \).

Give a greedy algorithm to assign the minimum number of people.

Challenge Question. This is for fun and enrichment only. Do not hand it in.

1. A generalization of the interval selection problem. Suppose that, instead of segments on a line, you are given subarcs of a circle, as in the figure below. Give an efficient algorithm to find a maximum size subset of the given arcs such that are pairwise disjoint.