# Lecture 11: Database Design (Theory)

CS348 Spring 2025:
Introduction to Database Management

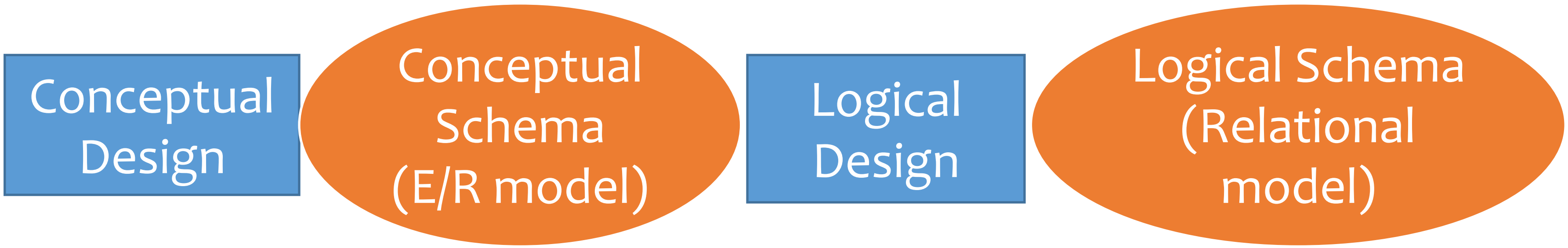
Instructor: **Xiao Hu**

Sections: 001, 002, 003

# Announcements

- Solutions of Assignment 1 will be released on Learn soon
- Grading of Assignment 1 will be released soon
- Appeal period of Assignment 1
  - One week after the grading is released
  - Watch out for the Piazza announcement!
  - Reach out to IA (Guy Coccimiglio) and the corresponding TA

# Database Design – where are we?




- Understand the real-world domain being modeled and constrained
- Entity-Relationship model
- Translate E/R diagram to relational data model
- (Refine a good database schema)
- Create DBMS schema (using DDL SQL)

# Case Study

- Consider a simple university DB:

Instructors

Departments

Courses

Students

- External application constraints such as:
  - Each instructor has name, salary, and department
  - Each instructor is officially affiliated with one department
  - Each department has one building and one budget
  - Each student can have at most one advisor from each department

# Case Study

Redundant data replication! (CS, DC, 20000) repeated k times if there are k instructors in CS!

- Possible Design: one large table InstructorDep with one row for each instructor

| instructorID | name | salary | depName | bldng | budget |
|---|---|---|---|---|---|
| 111 | Alice | 5000 | CS | DC | 20000 |
| 222 | Bob | 4000 | Physics | PHY | 30000 |
| 333 | Carl | 5200 | CS | DC | 20000 |
| 444 | Diana | 5500 | CS | DC | 20000 |

Fail to capture corner cases!

- If the building of CS is changed to E4?
- If the only instructor in Physics retires?
- If new department (w/o yet an instructor) is added?

# Case Study

- Possible Design: consider the following schema for courses with one row for each course offering

| CourseID | term | instructorName | capacity |
|---|---|---|---|
| CS348 | S23 | Sujaya | 100 |
| CS341 | W25 | Lap Chi | 80 |
| CS348 | W25 | Semih | 100 |
| CS348 | S25 | Xiao | 100 |
| CS350 | W19 | Salem | 130 |

- Is there any redundancy? Unclear!
    - Depends on the external application constraints
    - If courses have one associated capacity (independent of term): Redundant
    - Otherwise, repetition may be necessary

# Decompositions: A good example

Break down a complex database schema into smaller, more manageable pieces

| instructorID | name | salary | depName | bldng | budget |
|---|---|---|---|---|---|
| 111 | Alice | 5000 | CS | DC | 20000 |
| 222 | Bob | 4000 | Physics | PHY | 30000 |
| 333 | Carl | 5200 | CS | DC | 20000 |
| 444 | Diana | 5500 | CS | DC | 20000 |

| instructorID | name | salary | depName |
|---|---|---|---|
| 111 | Alice | 5000 | CS |
| 222 | Bob | 4000 | Physics |
| 333 | Carl | 5200 | CS |
| 444 | Diana | 5500 | CS |

$\bowtie$

| depName | bldng | budget |
|---|---|---|
| CS | DC | 20000 |
| Physics | PHY | 30000 |

| instructorID | name | salary | depName | bldng | budget |
|---|---|---|---|---|---|
| 111 | Alice | 5000 | CS | DC | 20000 |
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**Why can we recover all original tuples by joins?**

# Decompositions: A bad example

| <u>instructorID</u> | name | salary | depName | bldng | budget |
|---|---|---|---|---|---|
| 111 | Alice | 5000 | CS | DC | 20000 |
| 222 | Bob | 4000 | Physics | PHY | 30000 |
| 333 | Carl | 5200 | CS | DC | 20000 |
| 444 | Diana | 5500 | CS | DC | 20000 |

| instructorID |
|---|
| 111 |
| 222 |
| 333 |
| 444 |

⋈

| name |
|---|
| Alice |
| Bob |
| Carl |
| Diana |

⋈

| salary |
|---|
| 5000 |
| 4000 |
| 5200 |
| 5500 |

⋈

| depName |
|---|
| CS |
| Physics |

⋈

| bldng |
|---|
| DC |
| PHY |

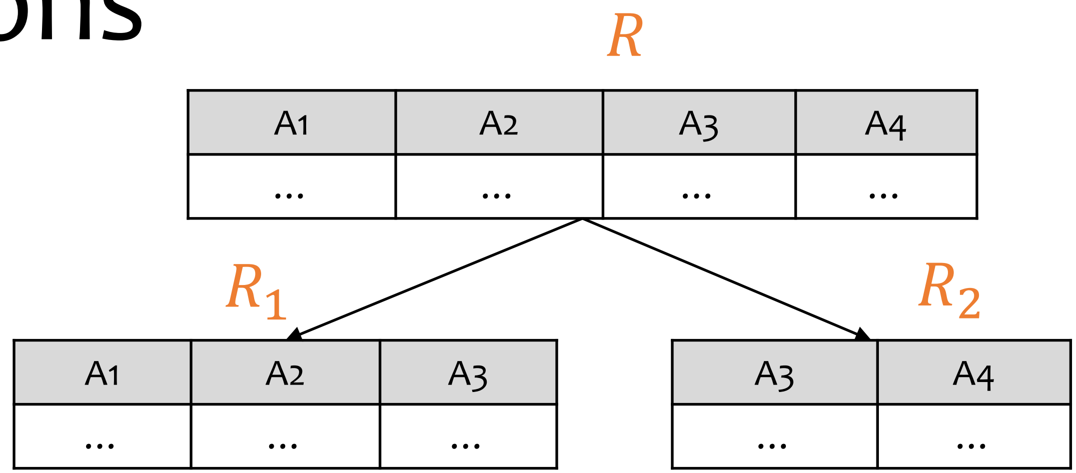
⋈

| budget |
|---|
| 20000 |
| 30000 |

| <u>instructorID</u> | name | salary | depName | bldng | budget |
|---|---|---|---|---|---|
| 111 | Alice | 5000 | CS | DC | 20000 |
| 111 | Bob | 5200 | CS | PHY | 30000 |
| … | … | … | … | … | … |

**Lossy? But I got more rows! Can't tell what's fact and what's not, so we lose information!**

# Decompositions




- $R$ is decomposed into $R_1$ and $R_2$
  - Attribute $(R_1)$ ∪ Attribute$(R_2)$ = Attribute $(R)$
  - $R_1$ and $R_2$ are the projections of $R$ onto Attribute $(R_1)$ and Attribute$(R_2)$
- Any decomposition gives $R \subseteq R_1 \bowtie R_2$
  - Lossless decomposition if $R = R_1 \bowtie R_2$
  - Lossy decomposition if $R \subset R_1 \bowtie R_2$

For any tuple $(a, b, c, d) \in R$, $(a, b, c) \in R_1$ and $(c, d) \in R_2$; then $(a, b, c) \bowtie (c, d) \in R_1 \bowtie R_2$

# Decompositions

- Break down a complex database schema into smaller, more manageable pieces while preserving data integrity and relationships

- What is a good or bad decomposition?

Normal Forms!

- How to obtain a good decomposition?

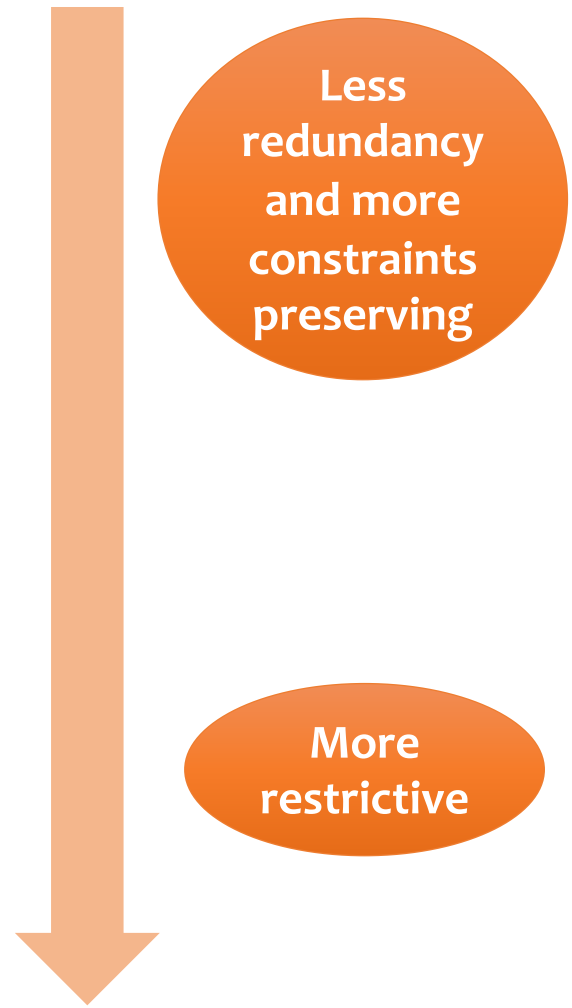
# Normal Forms

- Given a set of constraints about the real-world facts that an application will store, how can we formally separate "good" and "bad" relational database schemas?

- Normal Forms (NF): Normalization helps in reducing data redundancy and improving data integrity, making it easier to manage and maintain databases.

# Overview of Normal Forms

- First Normal Form (1NF)
  - atomic, domain
- Second Normal Form (2NF)
  - 1NF + no partial dependency
- Third Normal Form (3NF)
  - 2NF + no transitive dependency
- Boyce-Codd Normal Form (BCNF)
  - 3NF + dependency starts from the superkey
- Fourth Normal Form (4NF)
  - BCNF + no multi-valued dependency
- Fifth Normal Form (5NF)
  - 4NF + no redundancy due to join
- Sixth Normal Form (6NF)
  - 5NF + support temporal data



# What is next?

- Functional dependency (this lecture)
- Boyce-Codd Normal Form (BCNF)
- Third Normal Form (3NF)

# A Motivation Example

- Consider the following schema for InstructorDept

| <u>instructorID</u> | name | salary | depName | bldng | budget |
|---|---|---|---|---|---|

- Each instructorID has 1 name and salary
  - instructorID determines name and salary
- Each depName has 1 building and 1 associated budget
  - depName determines bldng and budget
- Each instructorID, depName is unique in InstructorDep
  - instructorID and depName together determine all remaining attributes, including name, salary, bldgn and budget
- How about instructorID and name together determining name? This is trivial!

# Functional dependencies

- $X \rightarrow Y$ means that whenever two tuples in $R$ agree on all the attributes in $X$, they must also agree on all attributes in $Y$

| $X$ | $Y$ | $Z$ |
|---|---|---|
| $a$ | $b$ | $c$ |
| $a$ | $b$ | ? |
| … | … | … |

Must be $b$

Could be anything

# Formal definition

Let t[A] be a tuple t's projection on attributes A

Let X, Y be sets of attributes

- A FD X $\longrightarrow$ Y holds in a relation R if any given pair of tuples $t_1$ and $t_2 \in$ R with $t_1[X] = t_2[X]$, we must have $t_1[Y] = t_2[Y]$.

- We say X determines Y
- A FD X $\longrightarrow$ Y holds in a relation R means that $X \rightarrow Y$ holds on all instances of $R$

# Redefining "keys" using FD's

A set of attributes *K* is a key for a relation R if

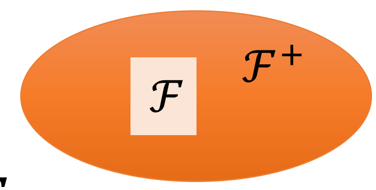
- *K* $\longrightarrow$ all (other) attributes of R
  - That is, K is a "super-key"
- No proper subset of *K* satisfies the above condition
  - That is, K is minimal

# Closure of FD sets: $\mathcal{F}^+$

- How do we know what additional FDs hold on a schema $R$?

- A set $\mathcal{F}$ of FDs logically implies $X \rightarrow Y$ if $X \rightarrow Y$ holds in all instances of $R$ that satisfy $\mathcal{F}$

- The closure of a FD set $\mathcal{F}$ (denoted as $\mathcal{F}^+$):
  - The set of all FDs that are logically implied by $\mathcal{F}$
  - Informally, $\mathcal{F}^+$ includes all of the FDs in $\mathcal{F}$, i.e., $\mathcal{F} \subseteq F^+$, plus any dependencies they imply.



# Armstrong's Axioms

- Reflexivity: If $Y \subseteq X$, then $X \rightarrow Y$ (trivially)

instructorID, name → instructorID

- Augmentation: if $X \rightarrow Y$, then $XZ \rightarrow YZ$ (trivially)

If instructorID → salary , then
instructorID, name → salary, name

- Transitivity: if $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

If instructor ID → depName and depName → budget,
then instructorID → budget

# Implications of Armstrong's Axioms

- Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$
- Union: If $X \rightarrow Y$ and $X \rightarrow Z$ then $X \rightarrow YZ$
- Pseudo-transitivity: If $X \rightarrow Y$ and $YZ \rightarrow T$ then $XZ \rightarrow T$
- Using Armstrong's Axioms, you can prove or disprove a (derived) FD given a set of (base) FDs

# Prove a FD in $\mathcal{F}^+$

$\mathcal{F}$ includes:

instructorID $\rightarrow$ name
projID $\rightarrow$ projName, projDep
instructorID, projID $\rightarrow$ hours
projDep, hours $\rightarrow$ funds

instructorID, projID $\rightarrow$ funds

- projID $\rightarrow$ projName, projDep
- projID $\rightarrow$ projDep (decomposition)
- instructorID, projID $\rightarrow$ instructorID, projDep (augmentation)
- instructorID, projID $\rightarrow$ hours
- instructorID, projID $\rightarrow$ instructorID, hours, projDep (union)
- instructorID, projID $\rightarrow$ hours, projDep (decomposition)
- hours, projDep $\rightarrow$ funds
- instructorID, projID $\rightarrow$ funds (transitivity)

# Compute $\mathcal{F}^+$ from $\mathcal{F}$

- Start with closure $\mathcal{F}^+ = \mathcal{F}$
- For each FD f in $\mathcal{F}^+$
  - Apply reflexivity and augmentation rules on f
  - Add the resulting FD to $\mathcal{F}^+$
- For each pair of FDs f1 and f2 in $\mathcal{F}^+$
  - If f1 and f2 can be combined using the transitivity rule, add the resulting FD to $\mathcal{F}^+$
- Repeat until no new FD can be added to $\mathcal{F}^+$

# Reasoning with $\mathcal{F}^+$

Given a relation $R$ and a set $\mathcal{F}$ of FD's

- Does another FD $X \rightarrow Y$ follow from $\mathcal{F}$?
  - Compute $\mathcal{F}^+$ with respect to $\mathcal{F}$
  - If $(X \rightarrow Y) \in \mathcal{F}^+$, then $X \rightarrow Y$ follows from $\mathcal{F}$
- Is $K$ a key of $R$?
  - If $(K \rightarrow R) \in \mathcal{F}^+$, $K$ is a super key
  - Still need to verify that $K$ is *minimal* (how?)
    - Hint: For any proper subset $X$ of $K$, $(X \rightarrow R) \notin \mathcal{F}^+$

# Attribute closure: $Z^+$

Given the relation schema $R$ and a set $\mathcal{F}$ of FDs

- The closure of attributes $X$ (denoted as $X^+$) is the set of all attributes $\{A_1, A_2, \ldots A_k\}$ functionally determined by $X$ (that is, $X \rightarrow A_1 A_2 \ldots \mathrm{A_K}$)

- Algorithm for computing the closure Compute$X^+(X, \mathcal{F})$:
  - Start with closure $= X$
  - If $Z \rightarrow Y$ is in $\mathcal{F}$ and $Z$ is already in the closure, then also add $Y$ to the closure
  - Repeat until no new attributes can be added

# In class exercise

- Given a relation R(ABCDEFG) under a set $\mathcal{F}$ of FDs, compute $X^+(\{B, F\}, \mathcal{F})$?

$\mathcal{F}$ includes:

$A, B \rightarrow F$

$A \rightarrow C$

$B \rightarrow E, D$

$D, F \rightarrow G$

| FD | $Z^+$ |
|---|---|
| initial | $B, F$ |
| $B \rightarrow E, D$ | $B, F, E, D$ |
| $D, F \rightarrow G$ | $B, F, E, D, G$ |

# In class exercise

- Given a relation EmpProj (SIN, pnum, hours, ename, pname, ploc, allowance) under a set $\mathcal{F}$ of FDs, compute$X^+$({pnum, hours}, $\mathcal{F}$)?

$\mathcal{F}$ includes:

*SIN, pnum → hours*
*SIN → ename*
*pnum → pname, ploc*
*ploc, hours → allowance*

| FD | $Z^+$ |
|---|---|
| initial | *pnum, hours* |
| *pnum → pname, ploc* | *pnum, hours, pname, ploc* |
| *ploc, hours → allowance* | *pnum, hours,pname, ploc, allowance* |

# In class exercise

- Given a relation EmpProj (SIN, pnum, hours, ename, pname, ploc, allowance) under a set $\mathcal{F}$ of FDs, compute$X^+$({SIN, pnum}, $\mathcal{F}$)?

$\mathcal{F}$ includes:

*SIN, pnum → hours*
*SIN → ename*
*pnum → pname, ploc*
*ploc, hours → allowance*

| **FD** | $Z^+$ |
|---|---|
| initial | *SIN, pnum* |
| *SIN → ename* | *SIN, pnum , ename* |
| *pnum → pname, ploc* | *SIN, pnum , ename, pname, ploc* |
| *SIN, pnum → hours* | *SIN, pnum , ename, pname, ploc, hours* |
| *ploc, hours → allowance* | *SIN, pnum , ename, pname, ploc, hours, allowance* |

- Compute$X^+$({SIN, pnum, hours}, $\mathcal{F}$)?

# Reasoning with $X^+$

Given a relation $R$ and a set $\mathcal{F}$ of FD's

- Does another FD $X \rightarrow Y$ follow from $\mathcal{F}$?
  - Compute $X^+$ with respect to $\mathcal{F}$
  - If $Y \subseteq X^+$, then $X \rightarrow Y$ follows from $\mathcal{F}$

- Is $K$ a key of $R$?
  - Compute $K^+$ with respect to $\mathcal{F}$
  - If $K^+$ contains all the attributes of $R$, $K$ is a super key
  - Still need to verify that $K$ is *minimal* (how?)
    - Hint: check the attribute closure of its proper subset, i.e., Check that for no set X formed by removing attributes from $K$ is $K^+$the set of all attributes

# Alternative: Compute $\mathcal{F}^+$ from $X^+$

Subset-closure enumeration

- List every subset $X \subseteq R$
- Compute its attribute closure $X^+$ under $\mathcal{F}$
- Then all FDs $X \to Y$ with $Y \subseteq X^+$ lie in $\mathcal{F}^+$

# What is next?

- Functional dependencies
  - Armstrong's axioms
  - Closure of FDs
  - Closure of attributes

- Next lecture: Decomposition
  - Boyce-Codd Normal Form (BCNF)
  - Third Normal Form (3NF)