# Intro to the Relational Model

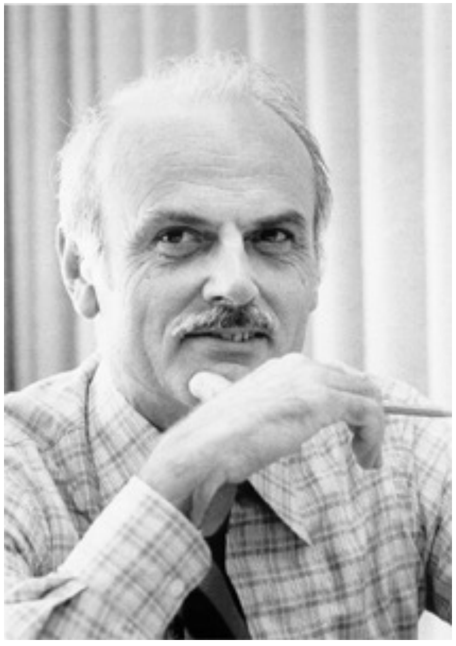
Introduction to Database Management

CS348 Fall 2022

# Announcements (Tue. Sep 13)

- Assignment #1 will be released on Learn 11pm today
  - Part 1: general questions and r.a.
    - Submit via Crowdmark
  - Part 2: writing SQL on DB2 on school servers (try soon)
    - Submit via Marmoset
  - Due by Sep 29 (Thur), 11:59pm
- Project details will be released on Learn on Thur
  - Grading scheme
  - Supplementary materials

# Edgar F. Codd (1923-2003)



- Pilot in the Royal Air Force in WW2
- Inventor of the relational model and algebra while at IBM
- Turing Award, 1981

http://en.wikipedia.org/wiki/File:Edgar_F_Codd.jpg

# Outline

- Part 1: Relational data model

- Part 2: Relational algebra

# Example for Relational data model

User

| uid | name | age | pop |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |
| 456 | Ralph | 8 | 0.3 |
| … | … | … | … |

Group

| gid | name |
|---|---|
| abc | Book Club |
| gov | Student Government |
| dps | Dead Putting Society |
| … | … |

Member

| uid | gid |
|---|---|
| 142 | dps |
| 123 | gov |
| 857 | abc |
| 857 | gov |
| 456 | abc |
| 456 | gov |
| … | … |

relations (or tables)

# Example for Relational data model

*User*

| *uid* | *name* | *age* | *pop* |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |
| 456 | Ralph | 8 | 0.3 |
| … | … | … | … |

*Group*

| *gid* | *name* |
|---|---|
| abc | Book Club |
| gov | Student Government |
| dps | Dead Putting Society |
| … | … |

*Member*

| *uid* | *gid* |
|---|---|
| 142 | dps |
| 123 | gov |
| 857 | abc |
| 857 | gov |
| 456 | abc |
| 456 | gov |
| … | … |

attributes (or columns)

# Example for Relational data model

*User*

| *uid* | *name* | *age* | *pop* |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |
| 456 | Ralph | 8 | 0.3 |
| … | … | … | … |

string int float

*Group*

| *gid* | *name* |
|---|---|
| abc | Book Club |
| gov | Student Government |
| dps | Dead Putting Society |
| … | … |

*Member*

| *uid* | *gid* |
|---|---|
| 142 | dps |
| 123 | gov |
| 857 | abc |
| 857 | gov |
| 456 | abc |
| 456 | gov |
| … | … |

domain (or type)

# Example for Relational data model

*User*

| *uid* | *name* | *age* | *pop* |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |
| 456 | Ralph | 8 | 0.3 |
| … | … | … | … |

*Group*

| *gid* | *name* |
|---|---|
| abc | Book Club |
| gov | Student Government |
| dps | Dead Putting Society |
| … | … |

tuples (or rows)

Duplicates are not allowed

Ordering of rows doesn't matter (even though output is always in some order)

*Member*

| *uid* | *gid* |
|---|---|
| 142 | dps |
| 123 | gov |
| 857 | abc |
| 857 | gov |
| 456 | abc |
| 456 | gov |
| … | … |

# Example for Relational data model

*User*

| *uid* | *name* | *age* | *pop* |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |
| 456 | Ralph | 8 | 0.3 |
| … | … | … | … |

*Group*

| *gid* | *name* |
|---|---|
| abc | Book Club |
| gov | Student Government |
| dps | Dead Putting Society |
| … | … |

*Member*

| *uid* | *gid* |
|---|---|
| 142 | dps |
| 123 | gov |
| 857 | abc |
| 857 | gov |
| 456 | abc |
| 456 | gov |
| … | … |

*User*: {⟨142, Bart, 10, 0.9⟩, ⟨857, Milhouse, 10, 0.2⟩, … }
*Group*: {⟨abc, Book Club⟩, ⟨gov, Student Government⟩, … }
*Member*: {⟨142, dps⟩, ⟨123, gov⟩, … }

# Relational data model

- A database is a collection of relations (or tables)
- Each relation has a set of attributes (or columns)
- Each attribute has a name and a domain (or type)
  - The domains are required to be atomic
- Each relation contains a set of tuples (or rows)
  - Each tuple has a value for each attribute of the relation
  - Duplicate tuples are not allowed
    - Two tuples are duplicates if they agree on all attributes

☞Simplicity is a virtue!

# Schema vs. instance

- Schema (metadata)
  - Specifies the logical structure of data
  - Is defined at setup time, rarely changes

*User* (*uid* int, *name* string, *age* int, *pop* float)
*Group* (*gid* string, *name* string)
*Member* (*uid* int, *gid* string)

- Instance
  - Represents the data content
  - Changes rapidly, but always conforms to the schema

*User*: {⟨142, Bart, 10, 0.9⟩, ⟨857, Milhouse, 10, 0.2⟩, … }
*Group*: {⟨abc, Book Club⟩, ⟨gov, Student Government⟩, … }
*Member*: {⟨142, dps⟩, ⟨123, gov⟩, … }

# Integrity constraints

- A set of rules that database instances should follow
- Example:
  - *age* cannot be negative
  - *uid* should be unique in the *User* relation
  - *uid* in *Member* must refer to a row in *User*

*User* (*uid* int, *name* string, *age* int, *pop* float)
*Group* (*gid* string, *name* string)
*Member* (*uid* int, *gid* string)

*User*: {⟨142, Bart, 10, 0.9⟩, ⟨857, Milhouse, 10, 0.2⟩, … }
*Group*: {⟨abc, Book Club⟩, ⟨gov, Student Government⟩, … }
*Member*: {⟨142, dps⟩, ⟨123, gov⟩, … }

# Integrity constraints

- An instance is only valid if it satisfies all the integrity constraints.
- Reasons to use constraints:
  - Ensure data entry/modification respects to database design
  - Protect data from bugs in applications

# Types of integrity constraints

- Tuple-level
  - Domain restrictions, attribute comparisons, etc.
    - E.g. *age* cannot be negative
- Relation-level
  - Key constraints (focus in this lecture)
    - E.g. *uid* should be unique in the *User* relation
  - Functional dependencies (Textbook, Ch. 7)
- Database-level
  - Referential integrity – foreign key (focus in this lecture)
    - *uid* in *Member* must refer to a row in *User* with the same *uid*

# Key (Candidate Key)

Def: A set of attributes $K$ for a relation $R$ if

- Condition 1: In no instance of $R$ will two different tuples agree on all attributes of $K$
  - That is, $K$ can serve as a "tuple identifier"
- Condition 2: No proper subset of $K$ satisfies the above condition
  - That is, $K$ is minimal

- Example: *User* (*uid*, *name*, *age*, *pop*)
  - *uid* is a key of *User*
  - *age* is not a key (not an identifier)
  - {*uid*, *name*} is not a key (not minimal), but a superkey

Satisfies only Condition 1

# Schema vs. instance

| *uid* | *name* | *age* | *pop* |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |
| 456 | Ralph | 8 | 0.3 |

- Is *name* a key of *User*?
  - Yes? Seems reasonable for this instance
  - No! User names are not unique in general
- Key declarations are part of the schema

# More examples of keys

- *Member* (*uid*, *gid*)
  - {*uid*, *gid*}
  - ☞A key can contain multiple attributes

*Member*

| *uid* | *gid* |
|---|---|
| 142 | dps |
| 123 | gov |
| 857 | abc |
| 857 | gov |
| 456 | abc |
| 456 | gov |
| … | … |

- *Address* (*street_address*, *city*, *state*, *zip*)
  - Key 1: {*street_address*, *city*, *state*}
  - Key 2: {*street_address*, *zip*}
  - ☞A relation can have multiple keys!

- Primary key: a designated candidate key in the schema declaration
  - Underline all its attributes, e.g., *Address* (<u>*street_address*</u>, *city*, *state*, <u>*zip*</u>)

# Use of keys

- More constraints on data, fewer mistakes
- Look up a row by its key value
  - Many selection conditions are “key = value”
- “Pointers” to other rows (often across tables)

# “Pointers” to other rows

- Foreign key: primary key of one relation appearing as attribute of another relation

*User*

| *uid* | *name* | *age* | *pop* |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |
| 456 | Ralph | 8 | 0.3 |
| … | … | … | … |

*Group*

| *gid* | *name* |
|---|---|
| abc | Book Club |
| gov | Student Government |
| dps | Dead Putting Society |
| … | … |

*Member*

| *uid* | *gid* |
|---|---|
| 142 | dps |
| 123 | gov |
| 857 | abc |
| 857 | gov |
| 456 | abc |
| 456 | gov |

# "Pointers" to other rows

- Referential integrity: A tuple with a non-null value for a foreign key that does not match the primary key value of a tuple in the referenced relation is not allowed.

*Group*

| gid | name |
|---|---|
| abc | Book Club |
| gov | Student Government |
| dps | Dead Putting Society |

*Member*

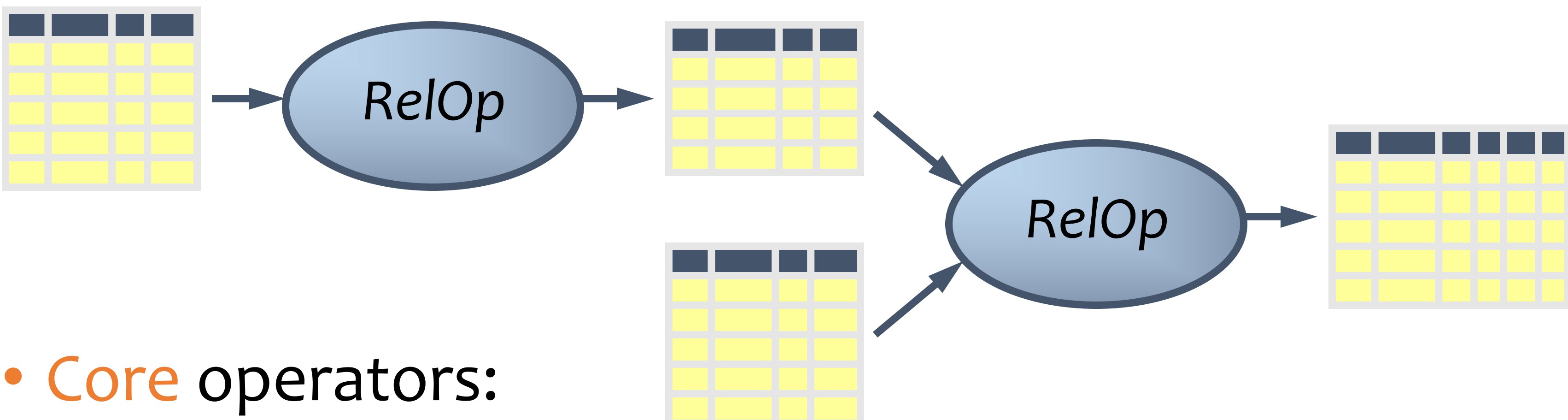
| uid | gid |
|---|---|
| 142 | dps |
| 123 | gov |
| 857 | ~~Xi~~ |
| 857 | gov |
| 456 | abc |
| 456 | gov |
| … | … |

# Outline

- Part 1: Relational data model
  - Data model
  - Database schema
  - Integrity constraints (keys)
  - Languages
    - Relational algebra (focus in this lecture)
    - SQL (next lecture)
    - Relational calculus (textbook, Ch. 27)
- Part 2: Relational algebra

# Relational algebra

A language for querying relational data based on “operators”




- Core operators:
  - Selection, projection, cross product, union, difference, and renaming
- Additional, derived operators:
  - Join, natural join, intersection, etc.
- Compose operators to make complex queries

# Core operator 1: Selection

- Example: Users with popularity higher than 0.5

$$\sigma_{pop>0.5} User$$

| *uid* | *name* | *age* | *pop* |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |
| 456 | Ralph | 8 | 0.3 |
| … | … | … | … |

→ $\sigma_{pop>0.5}$ →

| *uid* | *name* | *age* | *pop* |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
| [redacted] | | | |
| 857 | Lisa | 8 | 0.7 |
| [redacted] | | | |
| … | … | … | … |

# Core operator 1: Selection

- Input: a table $R$
- Notation: $\sigma_p R$
  - $p$ is called a selection condition (or predicate)
- Purpose: filter rows according to some criteria
- Output: same columns as $R$, but only rows of $R$ that satisfy $p$

# More on selection

- Selection condition can include any column of $R$, constants, comparison ($=$, $\leq$, etc.) and Boolean connectives (∧: and, ∨: or, ¬: not)
  - Example: users with popularity at least 0.9 and age under 10 or above 12

$$\sigma_{pop \geq 0.9 \wedge (age < 10 \vee age > 12)}\ User$$

- You must be able to evaluate the condition over each single row of the input table!
  - Example: the most popular user

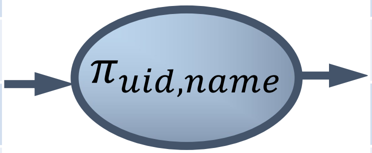
$$\sigma_{pop\ \geq\ every\ pop\ in\ User}\ User$$ **WRONG!**

# Core operator 2: Projection

- Example: IDs and names of all users

$$\pi_{uid,name}\ User$$

| *uid* | *name* | *age* | *pop* |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |
| 456 | Ralph | 8 | 0.3 |
| … | … | … | … |




| *uid* | *name* |
|---|---|
| 142 | Bart |
| 123 | Milhouse |
| 857 | Lisa |
| 456 | Ralph |
| … | … |

# Core operator 2: Projection

- Input: a table $R$
- Notation: $\pi_L R$
  - $L$ is a list of columns in $R$
- Purpose: output chosen columns
- Output: “same” rows, but only the columns in $L$

# More on projection

- Duplicate output rows are removed (by definition)
  - Example: user ages

$$\pi_{age}\, User$$

| uid | name | age | pop |
|---|---|---|---|
| 142 | Bart | 10 | 0.9 |
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |
| 456 | Ralph | 8 | 0.3 |
| … | … | … | … |

→ $\pi_{age}$ →

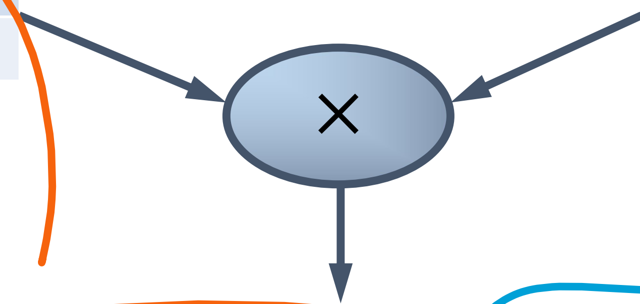
| age |
|---|
| 10 |
| [redacted] |
| 8 |
| [redacted] |
| … |

# Core operator 3: Cross product

$$User \times Member$$

| uid | name | age | pop |
|---|---|---|---|
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |

| uid | gid |
|---|---|
| 123 | gov |
| 857 | abc |
| 857 | gov |




| uid | name | age | pop | uid | gid |
|---|---|---|---|---|---|
| 123 | Milhouse | 10 | 0.2 | 123 | gov |
| 123 | Milhouse | 10 | 0.2 | 857 | abc |
| 123 | Milhouse | 10 | 0.2 | 857 | gov |
| 857 | Lisa | 8 | 0.7 | 123 | gov |
| 857 | Lisa | 8 | 0.7 | 857 | abc |
| 857 | Lisa | 8 | 0.7 | 857 | gov |

# Core operator 3: Cross product

- Input: two tables $R$ and $S$
- Natation: $R \times S$
- Purpose: pairs rows from two tables
- Output: for each row $r$ in $R$ and each $s$ in $S$, output a row $rs$ (concatenation of $r$ and $s$)

# A note a column ordering

- Ordering of columns is unimportant as far as contents are concerned

| *uid* | *name* | *age* | *pop* | *uid* | *gid* |
|---|---|---|---|---|---|
| 123 | Milhouse | 10 | 0.2 | 123 | gov |
| 123 | Milhouse | 10 | 0.2 | 857 | abc |
| 123 | Milhouse | 10 | 0.2 | 857 | gov |
| 857 | Lisa | 8 | 0.7 | 123 | gov |
| 857 | Lisa | 8 | 0.7 | 857 | abc |
| 857 | Lisa | 8 | 0.7 | 857 | gov |
| … | … | … | … | … | … |

=

| *uid* | *gid* | *uid* | *name* | *age* | *pop* |
|---|---|---|---|---|---|
| 123 | gov | 123 | Milhouse | 10 | 0.2 |
| 857 | abc | 123 | Milhouse | 10 | 0.2 |
| 857 | gov | 123 | Milhouse | 10 | 0.2 |
| 123 | gov | 857 | Lisa | 8 | 0.7 |
| 857 | abc | 857 | Lisa | 8 | 0.7 |
| 857 | gov | 857 | Lisa | 8 | 0.7 |
| … | … | … | … | … | … |

- So cross product is commutative, i.e., for any $R$ and $S$, $R \times S = S \times R$ (up to the ordering of columns)

# Derived operator 1: Join

- Info about users, plus IDs of their groups

$$User \bowtie_{User.uid=Member.uid} Member$$

| uid | name | age | pop |
|---|---|---|---|
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |
| … | … | … | … |

| uid | gid |
|---|---|
| 123 | gov |
| 857 | abc |
| 857 | gov |
| … | … |

$\bowtie$ $User.uid = Member.uid$

| uid | name | age | pop | uid | gid |
|---|---|---|---|---|---|
| 123 | Milhouse | 10 | 0.2 | 123 | gov |
| [redacted] | | | | | |
| 857 | Lisa | 8 | 0.7 | 857 | abc |
| 857 | Lisa | 8 | 0.7 | 857 | gov |
| … | … | … | … | … | … |

Prefix a column reference with table name and "." to disambiguate identically named columns from different tables

# Derived operator 1: Join

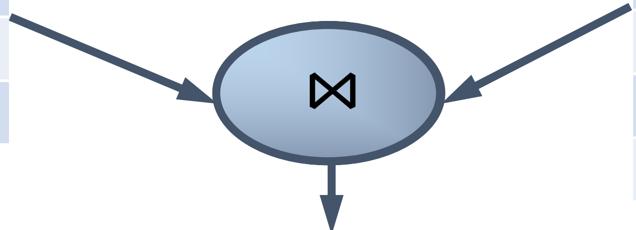
- Input: two tables $R$ and $S$
- Notation: $R \bowtie_p S$
  - $p$ is called a join condition (or predicate)
- Purpose: relate rows from two tables according to some criteria
- Output: for each row $r$ in $R$ and each row $s$ in $S$, output a row $rs$ if $r$ and $s$ satisfy $p$
- Shorthand for $\sigma_p(R \times S)$
- (A.k.a. “theta-join”)

# Derived operator 2: Natural join

$$User \bowtie Member$$
$$= \pi_{uid,name,age,pop,gid}\left(User \bowtie_{\substack{User.uid= \\ Member.uid}} Member\right)$$

| uid | name | age | pop |
|---|---|---|---|
| 123 | Milhouse | 10 | 0.2 |
| 857 | Lisa | 8 | 0.7 |
| … | … | … | … |

| uid | gid |
|---|---|
| 123 | gov |
| 857 | abc |
| 857 | gov |
| … | … |




| uid | name | age | pop | [redacted] | gid |
|---|---|---|---|---|---|
| 123 | Milhouse | 10 | 0.2 | [redacted] | gov |
| [redacted] | | | | | |
| 857 | Lisa | 8 | 0.7 | [redacted] | abc |
| 857 | Lisa | 8 | 0.7 | [redacted] | gov |
| … | … | … | … | [redacted] | … |

# Derived operator 2: Natural join

- Input: two tables $R$ and $S$
- Notation: $R \bowtie S$
- Purpose: relate rows from two tables, and
  - Enforce equality between identically named columns
  - Eliminate one copy of identically named columns
- Shorthand for $\pi_L(R \bowtie_p S)$, where
  - $p$ equates each pair of columns common to $R$ and $S$
  - $L$ is the union of column names from $R$ and $S$ (with duplicate columns removed)

# Core operator 4: Union

- Input: two tables $R$ and $S$
- Notation: $R \cup S$
  - $R$ and $S$ must have identical schema
- Output:
  - Has the same schema as $R$ and $S$
  - Contains all rows in $R$ and all rows in $S$ (with duplicate rows removed)

| uid | gid |
|---|---|
| 123 | gov |
| 857 | abc |

∪

| uid | gid |
|---|---|
| 123 | gov |
| 901 | edf |

=

| uid | gid |
|---|---|
| 123 | gov |
| 857 | abc |
| 901 | edf |

# Core operator 5: Difference

- Input: two tables $R$ and $S$
- Notation: $R - S$
  - $R$ and $S$ must have identical schema
- Output:
  - Has the same schema as $R$ and $S$
  - Contains all rows in $R$ that are not in $S$

| uid | gid |
|---|---|
| 123 | gov |
| 857 | abc |

$-$

| uid | gid |
|---|---|
| 123 | gov |
| 901 | edf |

$=$

| uid | gid |
|---|---|
| 857 | abc |

# Derived operator 3: Intersection

- Input: two tables $R$ and $S$
- Notation: $R \cap S$
  - $R$ and $S$ must have identical schema
- Output:
  - Has the same schema as $R$ and $S$
  - Contains all rows that are in both $R$ and $S$
- Shorthand for $R - (R - S)$
- Also equivalent to $S - (S - R)$
- And to $R \bowtie S$

# Core operator 6: Renaming

- Input: a table $R$
- Notation: $\rho_S\, R$, $\rho_{(A_1 \to A_1', \ldots)} R$, or $\rho_{S(A_1 \to A_1', \ldots)} R$
- Purpose: "rename" a table and/or its columns
- Output: a table with the same rows as $R$, but called differently

*Member*

| *uid* | *gid* |
|---|---|
| 123 | gov |
| 857 | abc |

$$\rho_{M1(uid \to uid_1, gid \to gid_1)} Member$$

*M1*

| *uid1* | *gid1* |
|---|---|
| 123 | gov |
| 857 | abc |

# 9. Basic operator: Renaming

- As with all other relational operators, it doesn't modify the database
  - Think of the renamed table as a copy of the original

- Used to
  - Create identical column names for natural joins
    - Example: $R(rid, \ldots), S(sid, \ldots.)$
    - $R \bowtie_{rid=sid} S$ can be written as $(\rho_{(rid \to id)} R) \bowtie (\rho_{sid \to id} S)$
  - Avoid confusion caused by identical column names

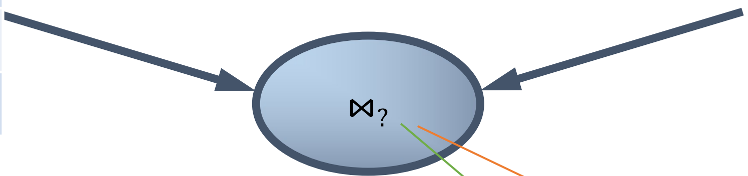
# 9. Basic operator: Renaming

- IDs of users who belong to at least two groups

$$Member \bowtie_{?} Member$$

| uid | gid |
|---|---|
| 100 | gov |
| 100 | abc |
| 200 | gov |

| uid | gid |
|---|---|
| 100 | gov |
| 100 | abc |
| 200 | gov |




| uid | gid | uid | gid |
|---|---|---|---|
| ~~100~~ | ~~gov~~ | ~~100~~ | ~~gov~~ |
| 100 | gov | 100 | abc |
| ~~100~~ | ~~gov~~ | ~~200~~ | ~~gov~~ |
| 100 | abc | 100 | gov |
| ~~100~~ | ~~abc~~ | ~~100~~ | ~~abc~~ |
| ~~100~~ | ~~abc~~ | ~~200~~ | ~~gov~~ |
| ~~200~~ | ~~gov~~ | ~~100~~ | ~~gov~~ |
| ~~200~~ | ~~gov~~ | ~~100~~ | ~~abc~~ |
| ~~200~~ | ~~gov~~ | ~~200~~ | ~~gov~~ |

Condition 1: same uid

Condition 2: different gids

# Renaming example

- IDs of users who belong to at least two groups

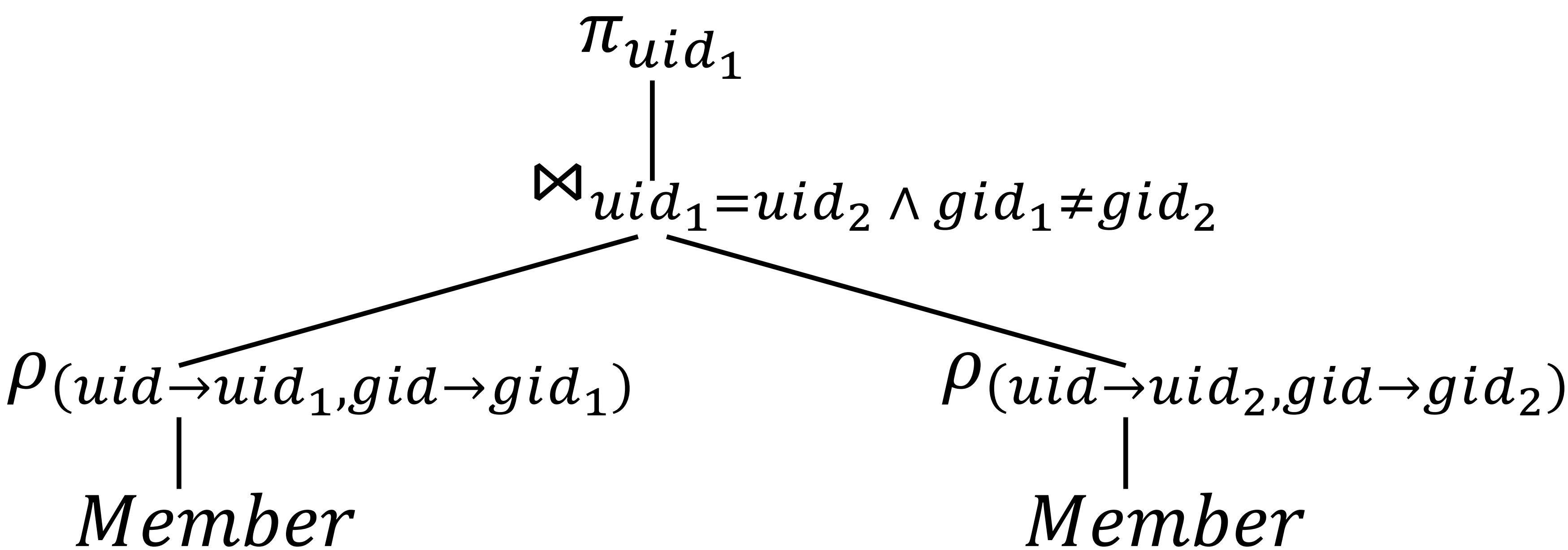
$$Member \bowtie_{?} Member$$

$$\pi_{uid}\left(Member \bowtie_{\substack{Member.uid=Member.uid \wedge \\ Member.gid \neq Member.gid}} Member\right)$$

**WRONG!**

$$\pi_{uid_1}\left(\begin{array}{c}\rho_{(uid \rightarrow uid_1, gid \rightarrow gid_1)} Member \\ \bowtie_{uid_1 = uid_2 \wedge gid_1 \neq gid_2} \\ \rho_{(uid \rightarrow uid_2, gid \rightarrow gid_2)} Member\end{array}\right)$$

# Expression tree notation



# Take-home Exercises

- Exercise 1: IDs of groups who have at least 2 users?
- Exercise 2: IDs of users who belong to at least three groups?

# Summary of operators

Core Operators

1. Selection: $\sigma_p R$
2. Projection: $\pi_L R$
3. Cross product: $R \times S$
4. Union: $R \cup S$
5. Difference: $R - S$
6. Renaming: $\rho_{S(A_1 \to A_1', A_2 \to A_2', \ldots)} R$

   Does not really add "processing" power

Note: Only use these operators for assignments & quiz

Derived Operators

1. Join: $R \bowtie_p S$
2. Natural join: $R \bowtie S$
3. Intersection: $R \cap S$

# More example

*User* (*uid* int, *name* string, *age* int, *pop* float)
*Group* (*gid* string, *name* string)
*Member* (*uid* int, *gid* string)

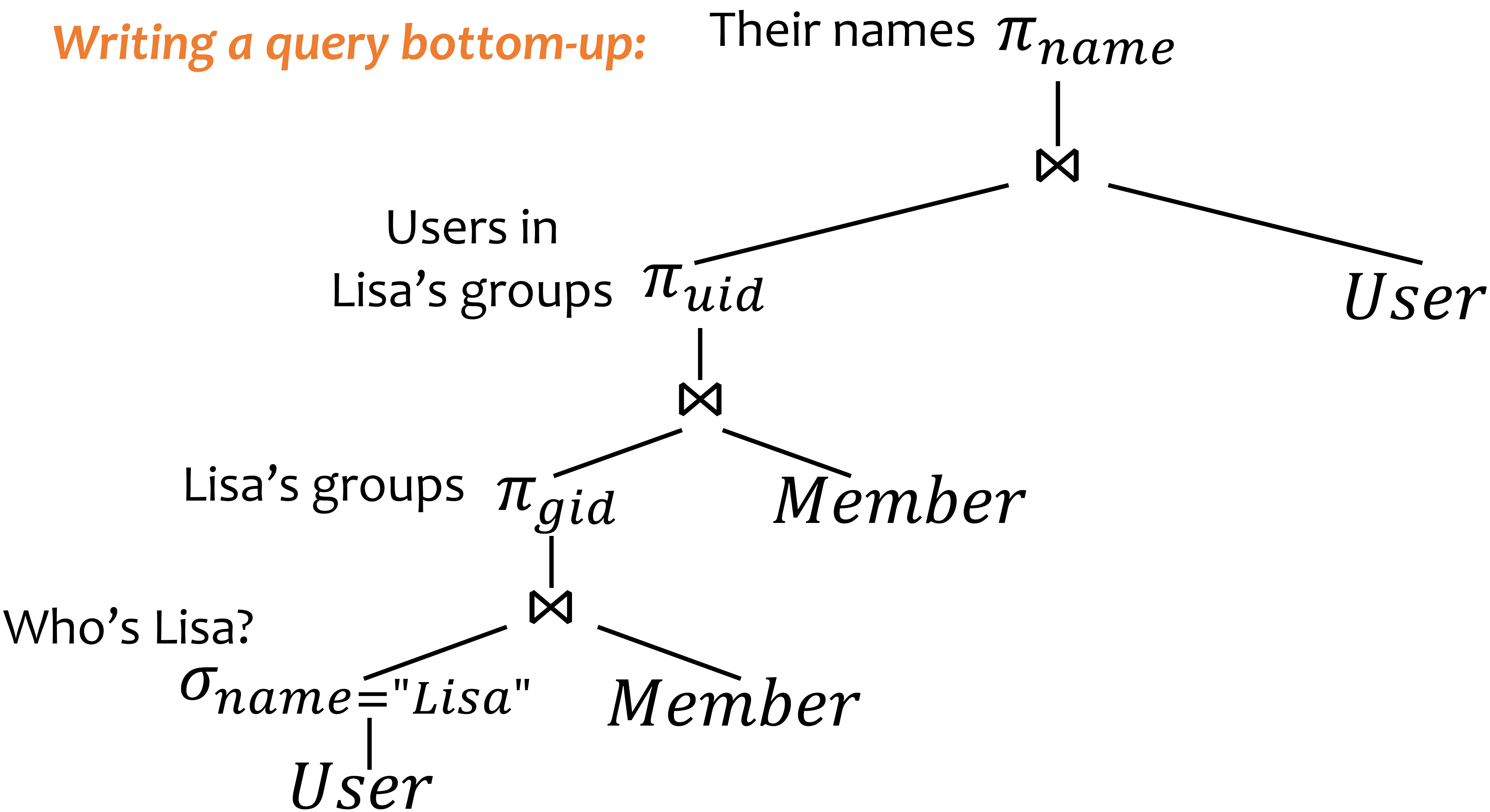
- Names of users in Lisa's groups

# More example

User (<u>uid</u> int, *name* string, *age* int, *pop* float)
Group (<u>gid</u> string, *name* string)
Member (<u>uid</u> int, <u>gid</u> string)

- Names of users in Lisa's groups

***Writing a query bottom-up:***



# More example

*User* (<u>*uid*</u> int, *name* string, *age* int, *pop* float)
*Group* (<u>*gid*</u> string, *name* string)
*Member* (<u>*uid*</u> int, <u>*gid*</u> string)

- IDs of groups that Lisa doesn’t belong to

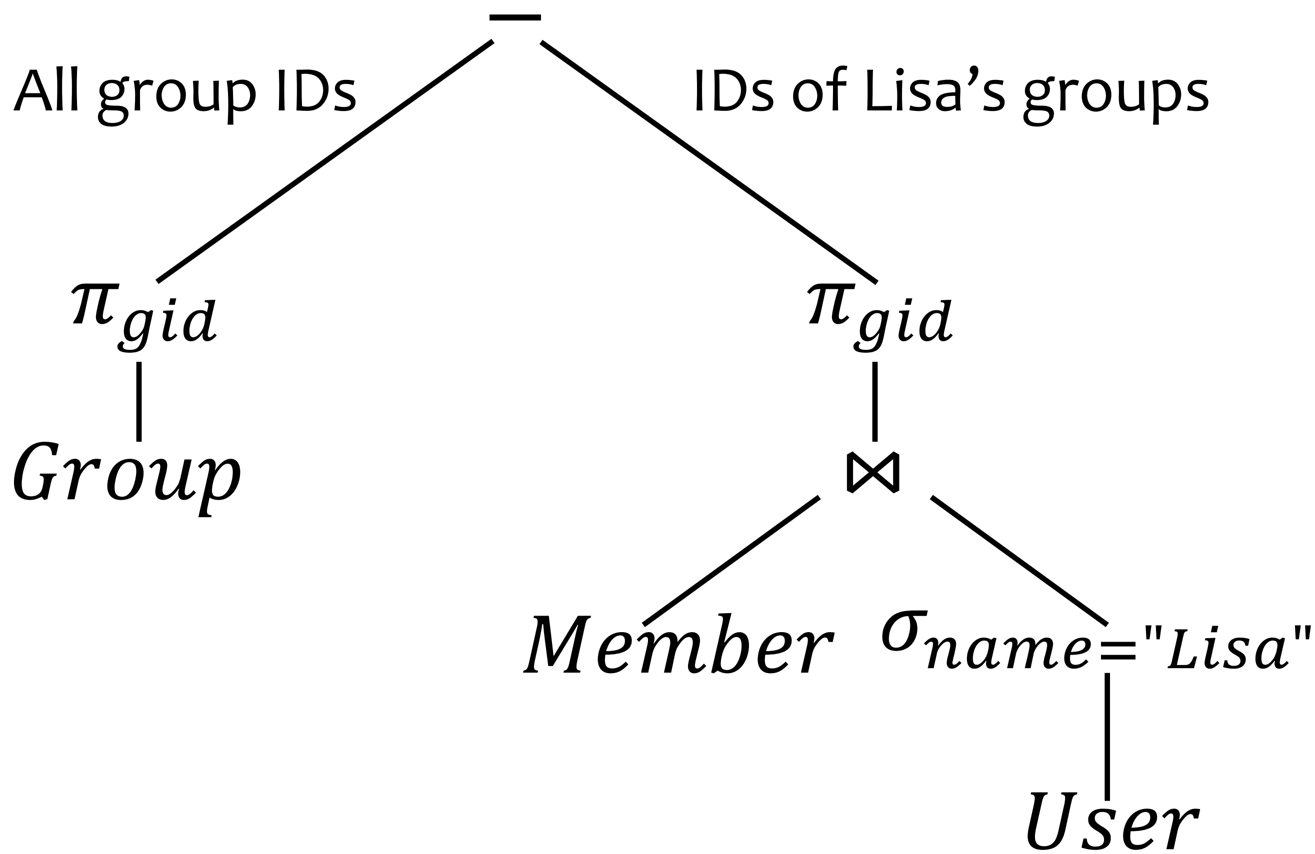
***Writing a query top-down:***

# More example

*User* (uid int, *name* string, *age* int, *pop* float)
*Group* (gid string, *name* string)
*Member* (uid int, gid string)

- IDs of groups that Lisa doesn't belong to

***Writing a query top-down:***



# A trickier example

*User* (<u>*uid*</u> int, *name* string, *age* int, *pop* float)
*Group* (<u>*gid*</u> string, *name* string)
*Member* (<u>*uid*</u> int, <u>*gid*</u> string)

- Who are the most popular?
    - Who do NOT have the highest *pop* rating?
    - Whose *pop* is lower than somebody else's?

# A trickier example

*User* (<u>*uid*</u> int, *name* string, *age* int, *pop* float)
*Group* (<u>*gid*</u> string, *name* string)
*Member* (<u>*uid*</u> int, <u>*gid*</u> string)

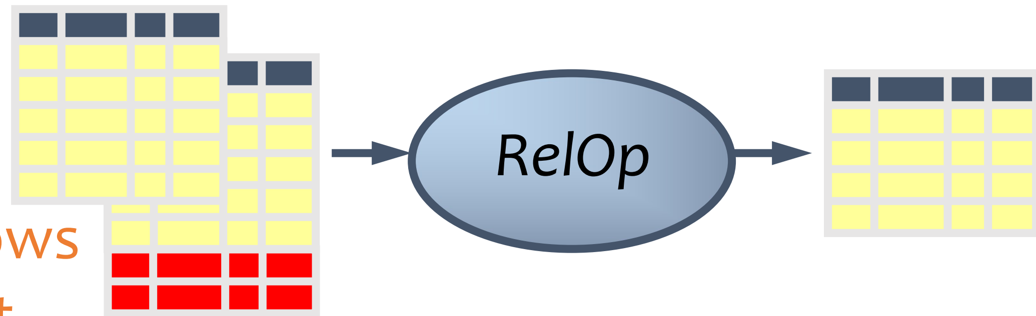
- Who are the most popular?
  - Who do NOT have the highest *pop* rating?
  - Whose *pop* is lower than somebody else's?

$$\pi_{uid} User - \pi_{User_1.uid}\left(\rho_{User_1} User \bowtie_{User_1.pop<User_2.pop} \rho_{User_2} User\right)$$

***A deeper question:***
***When (and why) is "—" needed?***

# Non-monotone operators

Add more rows to the input…




What happens to the output?

- If some old output rows may become invalid, and need to be removed → the operator is non-monotone

- Example: difference operator $R - S$

| uid | gid |
|---|---|
| 123 | gov |
| 857 | abc |

$R$

−

| uid | gid |
|---|---|
| 123 | gov |
| 901 | edf |
| **857** | **abc** |

$S$

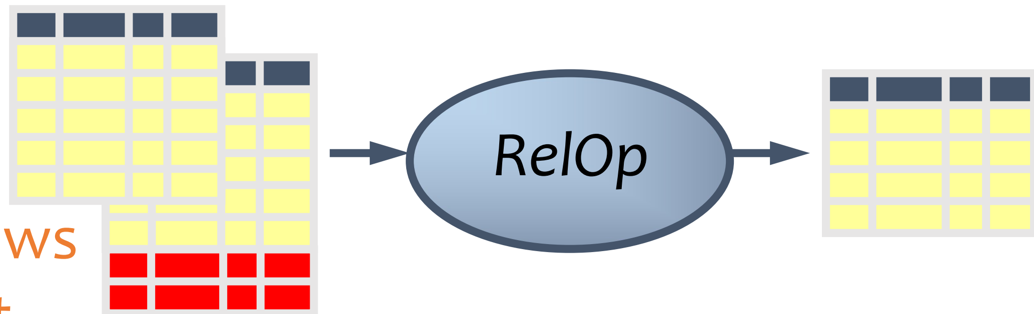
=

| uid | gid |
|---|---|
| ~~857~~ | ~~abc~~ |

This old row becomes invalid because the new row added to S

# Non-monotone operators

Add more rows to the input…




What happens to the output?

- If some old output rows may become invalid, and need to be removed → the operator is non-monotone
- Otherwise (old output rows always remain "correct") → the operator is monotone

*R*

| uid | gid |
|---|---|
| 123 | gov |
| 857 | abc |
| **189** | **abc** |

−

*S*

| uid | gid |
|---|---|
| 123 | gov |
| 901 | edf |

=

| uid | gid |
|---|---|
| 857 | abc |
| **189** | **abc** |

This old row is always valid no matter what rows are added to R

# Classification of relational operators

| | |
|---|---|
| • Selection: $\sigma_p R$ | Monotone |
| • Projection: $\pi_L R$ | Monotone |
| • Cross product: $R \times S$ | Monotone |
| • Join: $R \bowtie_p S$ | Monotone |
| • Natural join: $R \bowtie S$ | Monotone |
| • Union: $R \cup S$ | Monotone |
| • Difference: $R - S$ | Monotone w.r.t. $R$; non-monotone w.r.t $S$ |
| • Intersection: $R \cap S$ | Monotone |

# Why is "—" needed for "highest"?

- Composition of monotone operators produces a monotone query
  - Old output rows remain "correct" when more rows are added to the input
- Is the "highest" query monotone? (slide 50)
  - No!
  - Current highest *pop* is 0.9
  - Add another row with *pop* 0.91
  - Old answer is invalidated

☞So it must use difference!

# Why do we need core operator $X$?

- Difference
  - The only non-monotone operator
- Projection
  - The only operator that removes columns
- Cross product
  - The only operator that adds columns
- Union
  - ?
- Selection
  - ?

# Extensions to relational algebra

- Duplicate handling (“bag algebra”)
- Grouping and aggregation
- “Extension” (or “extended projection”) to allow new column values to be computed

☞All these will come up when we talk about SQL

☞But for now we will stick to standard relational algebra without these extensions

# Why is r.a. a good query language?

- Simple
  - A small set of core operators
  - Semantics are easy to grasp
- Declarative?
  - Yes, compared with older languages like CODASYL
  - Though operators do look somewhat “procedural”
- Complete?
  - With respect to what?

# Relational calculus

*User* (<u>*uid*</u> int, *name* string, *age* int, *pop* float)
*Group* (<u>*gid*</u> string, *name* string)
*Member* (<u>*uid*</u> int, <u>*gid*</u> string)

- First-order logic

- Example: Who are the most popular?
  - $\{u.uid \mid u \in User \wedge \neg(\exists u' \in User: u.pop < u'.pop)\}$, or
  - $\{u.uid \mid u \in User \wedge (\forall u' \in User: u.pop \geq u'.pop)\}$

# Relational calculus

- Relational algebra = "safe" relational calculus
  - Every query expressible as a safe relational calculus query is also expressible as a relational algebra query
  - And vice versa

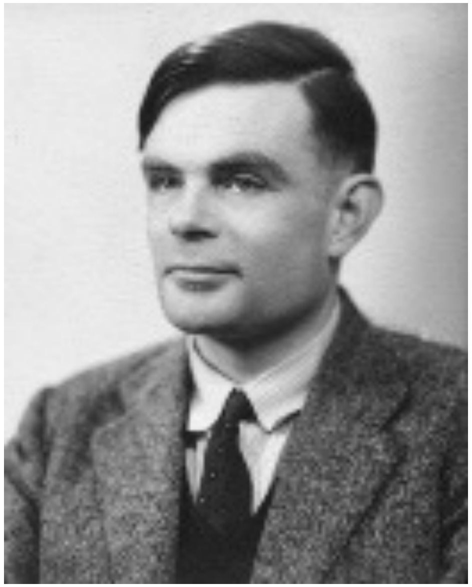
- Example of an "unsafe" relational calculus query
  - $\{u.name \mid \neg(u \in User)\}$ → users not in the database
  - Cannot evaluate it just by looking at the database

- A query is *safe* if, for all database instances conforming to the schema, the query result can be computed using only constants appearing in the database instance or in the query itself.

# Turing machine

How does relational algebra compare with a Turing machine?

- A conceptual device that can execute any computer algorithm
- Approximates what general-purpose programming languages can do
  - E.g., Python, Java, C++, …



Alan Turing (1912-1954)

http://en.wikipedia.org/wiki/File:Alan_Turing_photo.jpg

# Limits of relational algebra

- Relational algebra has no recursion
  - Example: given relation *Friend*(*uid1*, *uid2*), who can Bart reach in his social network with any number of hops?
    - Writing this query in r.a. is impossible!
  - So r.a. is not as powerful as general-purpose languages

- But why not?
  - Optimization becomes undecidable
  
  ☞Simplicity is empowering
  - Besides, you can always implement it at the application level, and recursion is added to SQL nevertheless!

# Summary

- Part 1: Relational data model
  - Data model
  - Database schema
  - Integrity constraints (keys)
  - Languages (relational algebra, relational calculus, SQL)

- Part 2: Relational algebra – basic language
  - Core operators & derived operators (how to write a query)
  - V.s. relational calculus
  - V.s. general programming language

- What's next?
  - SQL – query language used in practice (4 lectures)