SQL: Recursion (Optional)

Introduction to Database Management

CS348 Fall 2022

SQL

- Basic SQL (queries, modifications, and constraints)

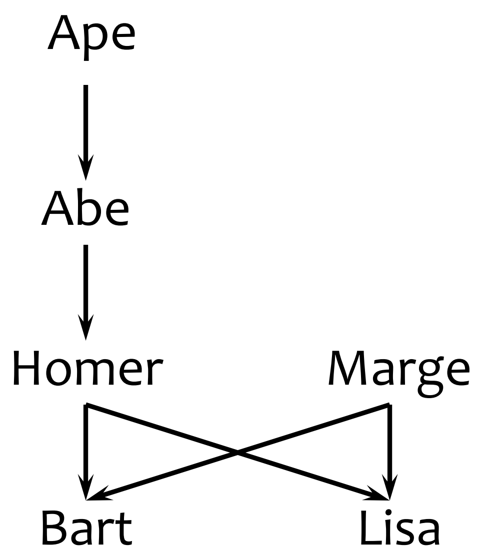
- Intermediate SQL
  - Triggers
  - Views
  - Indexes

- Advanced SQL
  - Programming
  - Recursion (Optional)

A motivating example

Parent (*parent*, *child*)

parent	*child*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe



- Example: find Bart's ancestors
- "Ancestor" has a recursive definition
  - X is Y's ancestor if
    - X is Y's parent, or
    - X is Z's ancestor and Z is Y's ancestor

Recursion in SQL

- SQL2 had no recursion
  - You can find Bart's parents, grandparents, great grandparents, etc.

```
SELECT p1.parent AS grandparent
FROM Parent p1, Parent p2
WHERE p1.child = p2.parent
    AND p2.child = 'Bart';
```

  - But you cannot find all his ancestors with a single query
- SQL3 introduces recursion
  - WITH clause
  - Implemented in PostgreSQL (common table expressions)

Ancestor query in SQL3

```
WITH RECURSIVE
Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
SELECT anc
FROM Ancestor
WHERE desc = 'Bart';
```

base case

a1.anc (X) → a1.desc(Z)
a2.anc (Z) → a2.desc (Y)

recursion step

Define a relation recursively

Query using the relation defined in WITH clause

Finding ancestors

```
WITH RECURSIVE
Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
.....;
```

base case

recursive step

parent	*child*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

anc	*desc*

anc	*desc*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

anc	*desc*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer

anc	*desc*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer
Ape	Bart
Ape	Lisa

Fixed point of a function

- If $f: D \to D$ is a function from a type D to itself, a fixed point of f is a value x such that $f(x) = x$
  - Example: what is the fixed point of f(x) = x/2?
  - Ans: 0, as f(0)=0 With seed 1: 1, 1/2, 1/4, 1/8, 1/16, … → 0

- To compute a fixed point of f
  - Start with a "seed": $x \leftarrow x_0$
  - Compute $f(x)$
    - If $f(x) = x$, stop; x is fixed point of f
    - Otherwise, $x \leftarrow f(x)$; repeat

Fixed point of a query

- A query q is just a function that maps an input table to an output table, so a fixed point of q is a table T such that $q(T) = T$

- To compute fixed point of q
  - Start with an empty table: $T \leftarrow \emptyset$
  - Evaluate q over T
    - If the result is identical to T, stop; T is a fixed point
    - Otherwise, let T be the new result; repeat

Non-linear v.s. linear recursion

- Non-linear

```
WITH RECURSIVE Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))  .....;
```

- Linear: a recursive definition can make only one reference to itself

```
WITH RECURSIVE Ancestor2(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT anc, child
  FROM Ancestor, Parent
  WHERE desc = parent))
```

Linear vs. non-linear recursion

- Linear recursion is easier to implement
  - For linear recursion, just keep joining newly generated *Ancestor* rows with *Parent*
  - For non-linear recursion, need to join newly generated *Ancestor* rows with all existing *Ancestor* rows

- Non-linear recursion may take fewer steps to converge, but perform more work
  - Example: Given $a \to b \to c \to d \to e$, *i.e.*, a is parent of b, b is parent of c, …., d is parent of e.
    - The linear recursion takes 4 steps to find (a, e) is an ancestor-descendant pair (slide 9, Ancestor2)
    - Question: How about non-linear recursion? (slide 9, Ancestor)

Mutual recursion example

- Table *Natural* (*n*) contains 1, 2, …, 100
- Which numbers are even/odd?
  - An even number plus 1 is an odd number
  - An odd number plus 1 is an even number
  - 1 is an odd number

```
WITH RECURSIVE Even(n) AS
    (SELECT n FROM Natural
     WHERE n = ANY(SELECT n+1 FROM Odd)),
RECURSIVE Odd(n) AS
    (SELECT n FROM Natural WHERE n = 1)
     UNION
    (SELECT n FROM Natural
     WHERE n = ANY(SELECT n+1 FROM Even
```

Computing mutual recursion

```
WITH RECURSIVE Even(n) AS
    (SELECT n FROM Natural
     WHERE n = ANY(SELECT n+1 FROM Odd)),
RECURSIVE Odd(n) AS
    ((SELECT n FROM Natural WHERE n = 1)
     UNION
     (SELECT n FROM Natural
      WHERE n = ANY(SELECT n+1 FROM Even
```

- *Even* = ∅, *Odd* = ∅
- *Even* = ∅, *Odd* = {1}
- *Even* = {2}, *Odd* = {1}
- *Even* = {2}, *Odd* = {1, 3}
- *Even* = {2, 4}, *Odd* = {1, 3}
- *Even* = {2, 4}, *Odd* = {1, 3, 5}
- …

Semantics of WITH

- WITH RECURSIVE R_1 AS Q_1,
  ...,
  RECURSIVE R_n AS Q_n
  Q;
  - Q and $Q_1, \ldots, Q_n$ may refer to $R_1, \ldots, R_n$
- Semantics
  1. $R_1 \leftarrow \emptyset, \ldots, R_n \leftarrow \emptyset$
  2. Evaluate $Q_1, \ldots, Q_n$ using the current contents of $R_1, \ldots, R_n$: $R_1^{new} \leftarrow Q_1, \ldots, R_n^{new} \leftarrow Q_n$
  3. If $R_i^{new} \neq R_i$ for some i
     3.1. $R_1 \leftarrow R_1^{new}, \ldots, R_n \leftarrow R_n^{new}$
     3.2. Go to 2.
  4. Compute Q using the current contents of $R_1, \ldots R_n$ and output the result

Starting with non-empty set

```
WITH RECURSIVE
Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
.....;
```

base case

recursive step

parent	*child*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

anc	*desc*
Bogus	**Bogus**

→

anc	*desc*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Bogus	**Bogus**

→

anc	*desc*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer
Bogus	**Bogus**

→

anc	*desc*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer
Ape	Bart
Ape	Lisa
Bogus	**Bogus**

Fixed points are not unique

```
WITH RECURSIVE
Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
.....;
```

parent	*child*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

anc	*desc*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer
Ape	Bart
Ape	Lisa
Bogus	Bogus

Lecture 2

Note how the bogus tuple reinforces itself!

- If q is monotone, then starting from Ø produces the unique minimal fixed point
  - All these fixed points must contain this fixed point

  → the unique minimal fixed point is the "natural" answer

Mixing negation with recursion

- If q is non-monotone
  - The fixed-point iteration may never converge
  - There could be multiple minimal fixed points

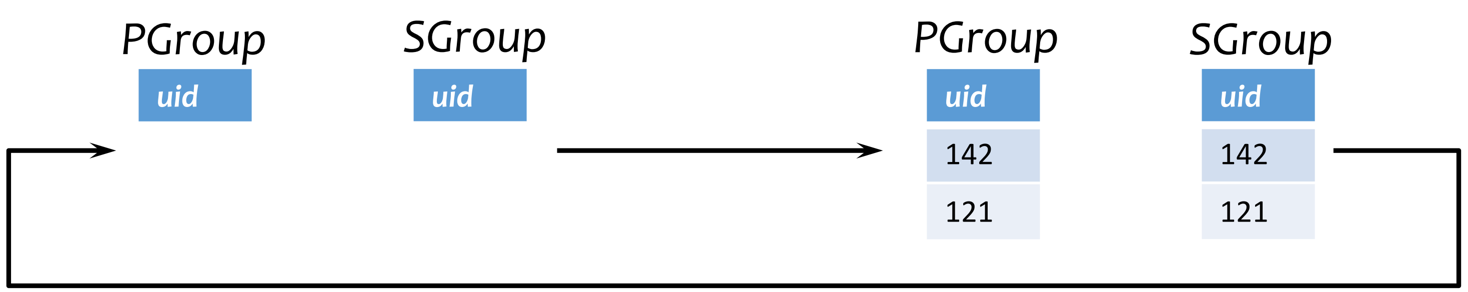
- Example: popular users (pop $\geq$ 0.8) join either SGroup or PGroup
  - Those not in SGroup should be in PGroup
  - Those not in GGroup should be in SGroup

```
WITH RECURSIVE PGroup(uid) AS
    (SELECT uid FROM User WHERE pop >= 0.8
     AND uid NOT IN (SELECT uid FROM SGroup)),
 RECURSIVE SGroup(uid) AS
    (SELECT uid FROM User WHERE pop >= 0.8
     AND uid NOT IN (SELECT uid FROM PGroup))
```

Fixed-point iter may not converge

```
WITH RECURSIVE PGroup(uid) AS
    (SELECT uid FROM User WHERE pop >= 0.8
     AND uid NOT IN (SELECT uid FROM SGroup)),
 RECURSIVE SGroup(uid) AS
    (SELECT uid FROM User WHERE pop >= 0.8
     AND uid NOT IN (SELECT uid FROM PGroup))
```

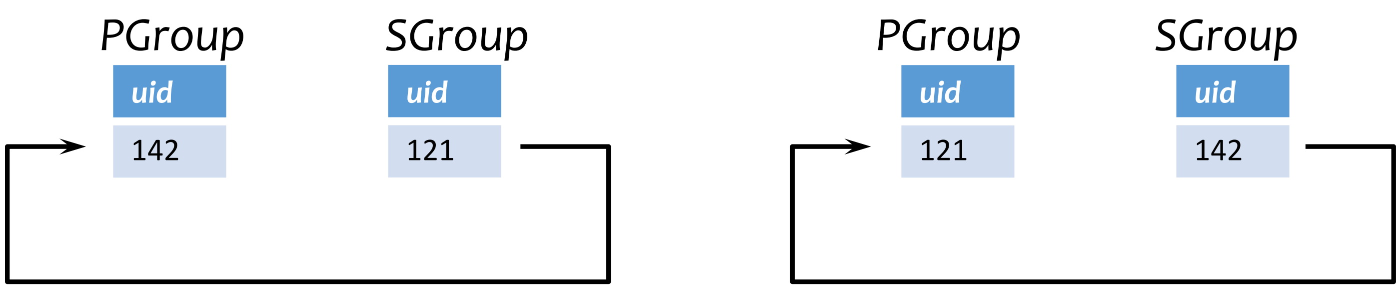
uid	*name*	*age*	*pop*
142	Bart	10	0.9
121	Allison	8	0.85



Multiple minimal fixed points

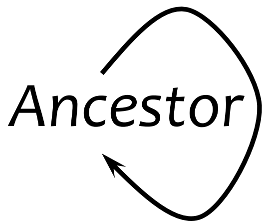
```
WITH RECURSIVE PGroup(uid) AS
    (SELECT uid FROM User WHERE pop >= 0.8
     AND uid NOT IN (SELECT uid FROM SGroup)),
 RECURSIVE SGroup(uid) AS
    (SELECT uid FROM User WHERE pop >= 0.8
     AND uid NOT IN (SELECT uid FROM PGroup))
```

uid	*name*	*age*	*pop*
142	Bart	10	0.9
121	Allison	8	0.85

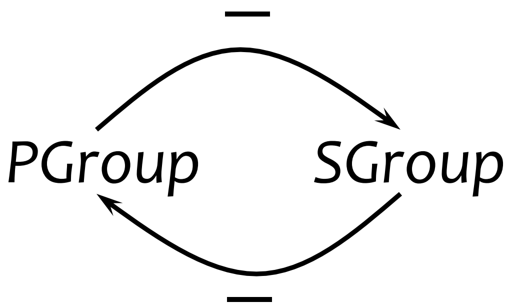


Legal mix of negation and recursion

- Construct a dependency graph
  - One node for each table defined in WITH
  - A directed edge $R \rightarrow S$ if R is defined in terms of S
  - Label the directed edge "–" if the query defining R is not monotone with respect to S
- Legal SQL3 recursion: no cycle with a "–" edge
  - Called stratified negation
- Bad mix: a cycle with at least one edge labeled "–"



Legal!



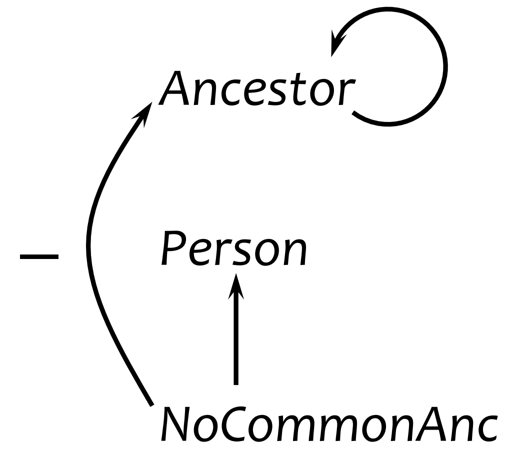
Illegal!

Stratified negation example

- Find pairs of persons with no common ancestors

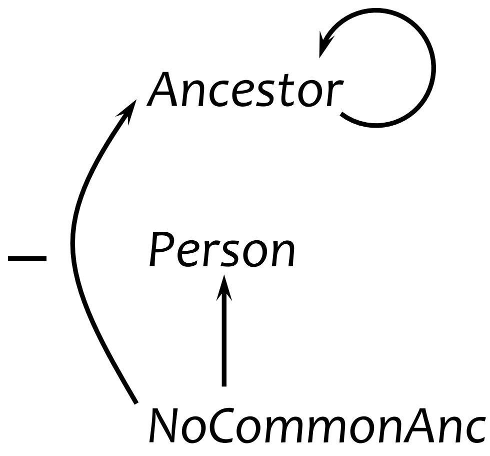
```
WITH RECURSIVE Ancestor(anc, desc) AS
    ((SELECT parent, child FROM Parent) UNION
     (SELECT a1.anc, a2.desc
      FROM Ancestor a1, Ancestor a2
      WHERE a1.desc = a2.anc)),
RECURSIVE  Person(person) AS
    ((SELECT parent FROM Parent) UNION
     (SELECT child FROM Parent)),
RECURSIVE NoCommonAnc(person1, person2) AS
    ((SELECT p1.person, p2.person
      FROM Person p1, Person p2
      WHERE p1.person <> p2.person)
     EXCEPT
     (SELECT a1.desc, a2.desc
      FROM Ancestor a1, Ancestor a2
      WHERE a1.anc = a2.anc))

SELECT * FROM NoCommonAnc;
```



Evaluating stratified negation

- The stratum of a node R is the maximum number of "—" edges on any path from R
  - *Ancestor*: stratum 0
  - *Person*: stratum 0
  - *NoCommonAnc*: stratum 1



- Evaluation strategy
  - Compute tables lowest-stratum first
  - For each stratum, use fixed-point iteration on all nodes in that stratum
    - Stratum 0: *Ancestor* and *Person*
    - Stratum 1: *NoCommonAnc*

☞ Intuitively, there is no negation within each stratum

Summary

- Basic SQL (queries, modifications, and constraints)
- Intermediate SQL(triggers, views, indexes)
- Programming

- Recursion
  - SQL3 WITH recursive queries
  - Solution to a recursive query (with no negation)
  - Mixing negation and recursion is tricky