- Start as early as possible, and contact the instructor if you get stuck.
- See the course outline for details about the course's marking policy and rules on collaboration.
- Submit your completed solutions to **Crowdmark**.

1. Context-Free Languages

[10] (a) Let $T_a = \{a, b\}$. Give a context-free grammar, G_a, which generates the **complement**, $(L_a)'$, of the language $L_a = \{a^n b^n \mid n \in 0, 1, 2, \ldots\}$ over T_a, and prove that your choice of G_a is correct.

[10] (b) Let $T_b = \{0, 1, !\}$. Give a context-free grammar, G_b, which generates the language

$$L_b = \{w!x \mid w, x \in \{0,1\}^* \text{ and } x \text{ contains } w^R \text{ as a substring}\},$$

over T_b, and prove that your choice of G_b is correct. (For example, $0111!11110 \in L_b$.)

2. Some context-free languages are regular

[8] Let $T = \{0,1\}$. Suppose a language $L \subseteq T^*$ is accepted by some PDA, P, by final state (i.e. $L = L(P)$). Suppose further that while processing any word $w \in T^*$, P's stack never contains more than 3 elements of the stack alphabet, Γ. Prove that L is **regular**.

3. Removing ambiguity in context-free grammars

   Define a context-free grammar $G = (V, T, P, S)$, where

   - $V = \{S\}$
   - $T = \{a, b\}$
   - $P = \{S \to aS | aSbS | \varepsilon\}$
   - $S = S$

[2] (a) Prove that G is **ambiguous**, by exhibiting two different parse trees, two different leftmost derivations or two different rightmost derivations for the word $aab \in L(G)$.

[8] (b) Prove that

$$L(G) = \{w \in T^* \mid \text{for every prefix } x \text{ of } w,\ n_a(x) \geq n_b(x)\}.$$

[12] (c) Exhibit (with proof) an unambiguous grammar, G', such that $L(G') = L(G)$.

4. A pushdown automaton

Define a pushdown automaton, $P = (Q, \Sigma, \Gamma, \delta, q, Z_0, F)$, accepting by final state, with

- $Q = \{q, p\}$
- $\Sigma = \{0, 1\}$
- $\Gamma = \{Z_0, X\}$
- q = start state for machine
- Z_0 = stack start letter (bottom of stack character)
- $F = \{p\}$

and transition function

$$\begin{aligned}
\delta(q, 0, Z_0) &= \{(q, XZ_0)\} \\
\delta(q, 0, X) &= \{(q, XX)\} \\
\delta(q, 1, X) &= \{(q, X)\} \\
\delta(q, \varepsilon, X) &= \{(p, \varepsilon)\} \\
\delta(p, \varepsilon, X) &= \{(p, \varepsilon)\} \\
\delta(p, 1, X) &= \{(p, XX)\} \\
\delta(p, 1, Z_0) &= \{(p, \varepsilon)\}
\end{aligned}$$

[2] (a) Draw a diagram for P.

[6] (b) Determine all the instantaneous descriptions of the machine P which can be reached after processing the input word $w = 01$.

[2] (c) Using the definition of acceptance by final state, is $01 \in L(P)$? Justify your answer.