University of Waterloo
Final Examination

Fall 2010

Student Name _____

Student ID Number _____

Course Abbreviation & Number	CS 360
Course Title	Theory of Computation
Section	01
Instructor	dan brown

Date of Exam	December 16	
Time Period: evening	Start Time: 7.30 pm	End Time: 10 pm
Duration of Exam:	150 minutes	
Number of Exam Pages (including this cover sheet)	11 pages	

1. Complete all answers in the spaces provided.
2. Write neatly so you do not lose marks unnecessarily.
3. Proctors will only confirm or deny the existence of errors on the exam.
4. In the case of perceived ambiguity, state a clear assumption and proceed to answer the question.
5. Cheating is an academic offence. Your signature on this exam indicates that you understand and agree to the university's policies regarding cheating on exams.

#	Marks	Actual	Initial
1	8		
2	8		
3	8		
4	8		
5	8		
6	8		
7	12		
$\sum$	60		

Signature:_____

1. (8 marks): **A context-free language**
   For $w \in \{0,1\}^*$, let $\tilde{w}$ be the word that results from replacing all 0s in w with 1s and all 1s with 0s. For example, if $w = 011$, then $\tilde{w} = 100$.

   Let $L = \{w!\tilde{w}^R \mid w \in \{0,1\}^*\}$. Show L is context free by giving a PDA that accepts L by final state. Explain informally why your PDA is correct.

2. (8 marks): **A regular language**
   Show that the language $L = \{a^i b^j \mid i \mod 3 = j \mod 3\}$ is regular by giving a DFA that accepts L. Explain informally why your DFA is correct.

3. (8 marks): **Short answers**

   (a) Is it always the case that the reverse of a decidable language is decidable? Prove your answer. (4 marks)

(b) Is it always the case that the union of a decidable language and a regular language is regular? Prove your answer. (4 marks)

4. (8 marks): **A non-context-free language**
   Consider the language $L = \{x \mid x \in \{0,1,2\}^*, n_0(x) \geq n_1(x), n_0(x) \leq n_2(x)\}$.

   Prove L is not context free.

5. (8 marks): **An undecidable language**
   Prove that the problem of determining whether a Turing machine and a regular expression have any words in their language in common is not decidable.

   Show this by giving a reduction, not by using Rice's theorem.

6. (8 marks): **A decidable language**
   Suppose that we have a way of encoding PDAs and regular expressions such that they can be the input to a Turing machine.

   Show that the problem of determining, given a PDA P and a regular expression E, if E generates any words in $L(P)$, is decidable by giving an algorithm for the problem and explaining why the algorithm is correct.

7. (12 marks): **Ambiguity in CFLs**
   Let L be the language of this grammar:

$$G : S \to aSb \mid aSbb \mid aaSb \mid \varepsilon$$

   (a) What is the language of this grammar? Explain informally why you are right. (4 marks)

(b) Show G is ambiguous. (2 marks)

(c) Is L inherently ambiguous? If so, explain convincingly why it is. If not, given an unambiguous grammar for L and explain why it is unambiguous and generates exactly L. (6 marks)