University of Waterloo
Midterm Examination

Spring 2013

Student Name	_____
Student ID Number	_____

Course Abbreviation & Number	CS 360
Course Title	Theory of Computation
Section	01
Instructor	dan brown

Date of Exam	July 3	
Time Period: in-class	Start Time: 12.30 pm	End Time: 1.20 pm
Duration of Exam:	50 minutes	
Number of Exam Pages (including this cover sheet)	6	

1. Complete all answers in the spaces provided.
2. Write neatly so you do not lose marks unnecessarily.
3. Proctors will only confirm or deny the existence of errors on the exam.
4. In the case of perceived ambiguity, state a clear assumption and proceed to answer the question.
5. Cheating is an academic offence. Your signature on this exam indicates that you understand and agree to the university's policies regarding cheating on exams.

#	Marks	Actual	Initial
1	8		
2	15		
3	10		
$\sum$	33		

Signature:_____

1. (8 marks): **A non-regular language**
   Consider the language $L = \{0^i 1^j 0^k 1^\ell \mid i + j = k + \ell \text{ and } i, j, k, \ell \geq 0\}$.

   Prove that L is not regular.

2. (15 marks): **A context-free language**
All four parts of this question concern the context-free grammar
$G : S \to 0S11 \mid 0S111 \mid \varepsilon$.

   (a) (2 marks) What is $L(G)$? (Describe the language; you'll prove your answer is correct in parts (b) and (c).)

   (b) (4 marks) Let L be the language you described in part (a). Prove that $L \subseteq L(G)$.

(c) (6 marks) Let L be the language you described in part (a). Prove that $L(G) \subseteq L$. G is still the grammar $G : S \to 0S11 \mid 0S111 \mid \varepsilon$.

(d) (3 marks) Prove that the grammar $G : S \to 0S11 \mid 0S111 \mid \varepsilon$ is ambiguous.

3. (10 marks): **A pushdown automaton**
   Draw a pushdown automaton that accepts the language $L = \{0^i 1^j 0^k 1^\ell \mid i + j = k + \ell$ and $i, j, k, l \geq 0\}$ by final state.

   L is the language you proved is not regular in Question 1.

   Explain briefly why your automaton is correct (you do not need to give a full proof).