1. [10 marks] We say a string $x$ is a cyclic shift of a string $w$ if there exist strings $u, v$ such that $x = uv$ and $w = vu$.

For example, enlist is a cyclic shift of listen.

Define $\text{cyc}(L)$ be the language of all cyclic shifts of all elements of $L$:

$$\text{cyc}(L) = \{ x : w \in L \text{ for some cyclic shift } w \text{ of } x \}. $$

(a) Prove that if $L$ is Turing-decidable, then so is $\text{cyc}(L)$.
(b) Is it true that if $L$ is Turing-recognizable, then so is $\text{cyc}(L)$?

2. [10 marks] Recall the substring operation on languages:

$$\text{subst}(L) = \{ x : \exists y \in L \text{ such that } x \text{ is a substring of } y \}. $$

Is the class of Turing-decidable languages closed under the operation $\text{subst}$? Prove or give a counterexample.

(It may be very helpful to read the course notes, Chapter 17.)

3. [10 marks] Prove that every infinite Turing-recognizable language has an infinite Turing-decidable subset.