1. [10 marks] Recall the example DFA from the Week 2 slides entitled “Deterministic Finite Automata” slide 25, available on the course home page.
Find a regular expression specifying the language recognized by this DFA. Show your work.

```
         0
q0 ---> 1 0 --> q1 0 --> q2
          1 0
```

2. [10 marks]
Use the pumping lemma to prove that the language
\[ L = \{0^n1xy : x, y \in \{0, 1\}^*, |x| = n \geq 0\} \]
is not regular.

3. [10 marks]
Suppose we define an operation on nonempty strings, as follows: even(x) deletes all symbols of x occurring at odd-numbered positions. More precisely, for \( n \geq 0 \) we have \( \text{even}(a_1a_2\cdots a_{2n+1}) = \text{even}(a_1a_2\cdots a_{2n}) = a_2a_4a_6\cdots a_{2n}. \)

Thus, for example, \( \text{even}(\text{connected}) = \text{once}. \)
Extend this to languages in the obvious way: \( \text{even}(L) = \{\text{even}(x) : x \in L\} \).

Prove that if \( L \) is regular then so is \( \text{even}(L) \).

Hint: start with a DFA for \( L \) and modify it somehow, getting an \( \epsilon \)-NFA for \( \text{even}(L) \).
In addition to giving your construction in detail, be sure to give a proof that it actually works.

This one is likely to be somewhat hard.