1. [10 marks] Draw the transition diagram of a Turing machine that doubles every letter of the input string $x \in \{a, b\}^*$. More precisely, if the Turing machine starts in a configuration $(q_0, \omega)a_1a_2\cdots a_n$ then it should end in a configuration $a_1a_1a_2a_2\cdots a_na_n(q_{\text{acc}}, \omega)$.

Use the basic Turing machine model in your solution (a single two-sided unbounded tape, left and right moves only, deterministic). There’s no need to give a formal proof of correctness, but explain your construction in a few lines.

Hint: to simplify your diagram, you can use a construction in https://student.cs.uwaterloo.ca/~cs360/tm4a.pdf as a subroutine.

2. [10 marks] Prove, with an explicit construction, that every regular language is Turing-decidable. Hint: create the transition function of a TM directly from the transition function of a DFA.

Use the basic Turing machine model in your solution (a single two-sided unbounded tape, left and right moves only, deterministic). There’s no need to give a formal proof of correctness, but explain your construction in a few lines.

3. [10 marks] Suppose that $L$ is a language that is Turing-recognizable but not Turing-decidable. Show that every TM $M$ recognizing $L$ must, on infinitely many different inputs, fail to halt.