Marking symbols

We already saw this technique in previous slides.

The idea is to change symbols by marking them, to keep track of progress during a computation.

When the computation is complete, the marks can be removed.

Marking a symbol takes place by using other letters from the tape alphabet.

For example, if the input alphabet is \{a, b\}, we could add symbols \(a'\) and \(b'\) to the tape alphabet, and use these as the “marked” versions of \(a\) and \(b\).
Consider the following problem: we want to count the number of $b$’s in an input string.

More precisely, what we want to do is the following: on input $x \in \{a, b\}^*$ we want the tape to end up with contents $x_{\bot}b^i$, where $i = |x|_b$.

How can we do this?

The idea is to scan the input from left to right.

If the symbol is a $b$, we mark it (by changing it to $b'$) and then move to the right, adding a $b$ to the list of $b$’s we’ve already accumulated at the end.
Marking symbols

Then we move left to the first marked symbol, unmark it, and continue.

This gives us the following TM:
Inserting a blank

Sometimes we need to “insert” some extra space on a tape, to allow room for additional computation.

We can do this with an “insert blank subroutine” that can be called as needed.

The idea is that the tape initially looks like $xcy$, with the tape head scanning the $c$ and $x, y$ are arbitrary strings (not containing blanks) and $c$ is a single nonblank symbol.

After the subroutine is complete the tape looks like $xc\_\_y$, and we’re again scanning the $c$.

So we have inserted a blank after the $c$ and the portion of the string appearing after $xc$ has been “slid down” one cell.
Inserting a blank

We are now free to insert a new letter in the blank position, if we want to.

If we still don’t have enough room for what we want to insert, we can just call the “insert blank subroutine” over and over.

Note that “calling a subroutine” really means “add these rules to the transition function of the TM, possibly changing the state names, if necessary”.

This gives the following TM.
Inserting a blank