Data-Intensive Distributed Computing

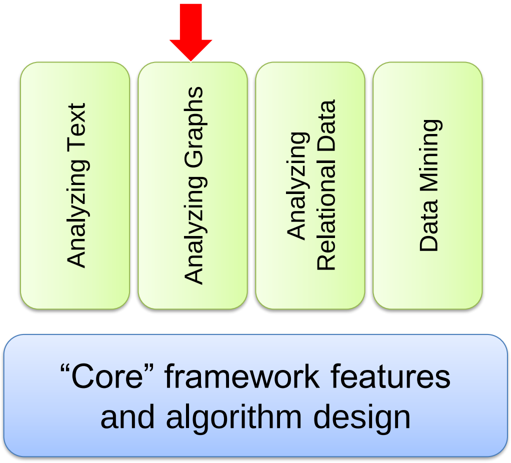
CS 431/631 451/651 (Fall 2021)

Part 5: Analyzing Graphs (1/2)

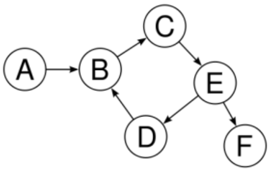
Ali Abedi

These slides are available at https://www.student.cs.uwaterloo.ca/~cs451/

Structure of the Course



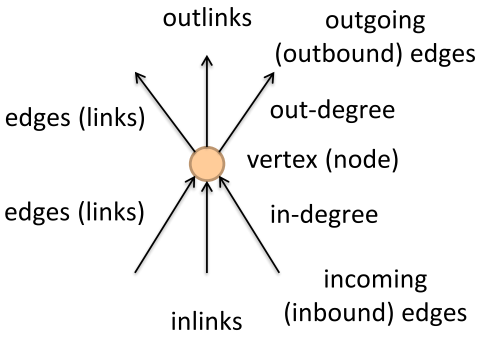
“Core” framework features and algorithm design



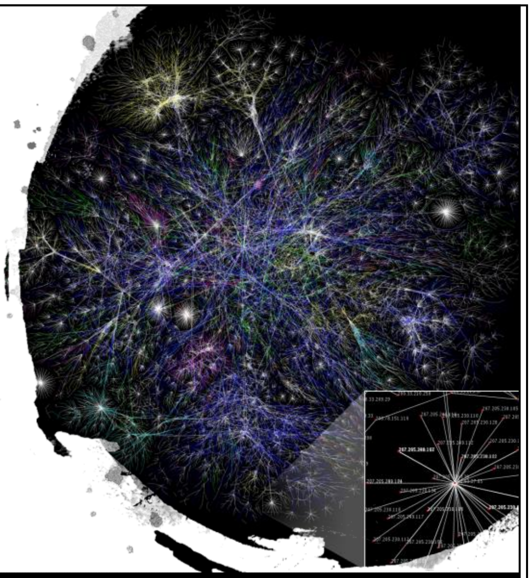
What’s a graph?

G = (V,E), where

V represents the set of vertices (nodes)
E represents the set of edges (links)
Edges may be directed or undirected
Both vertices and edges may contain additional information

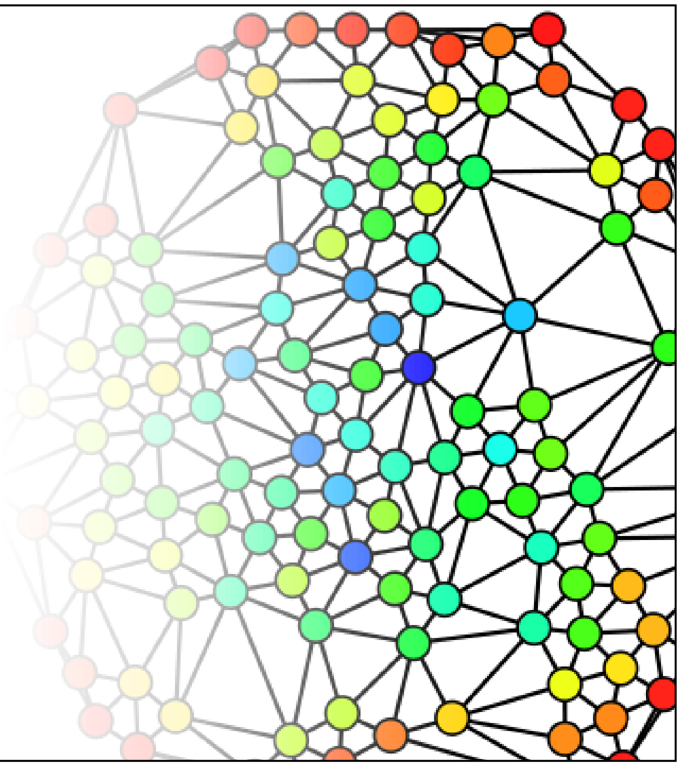

outlinks
outgoing
(outbound) edges
edges (links)
out-degree
vertex (node)
edges (links)
in-degree
incoming
(inbound) edges
inlinks

Examples of Graphs



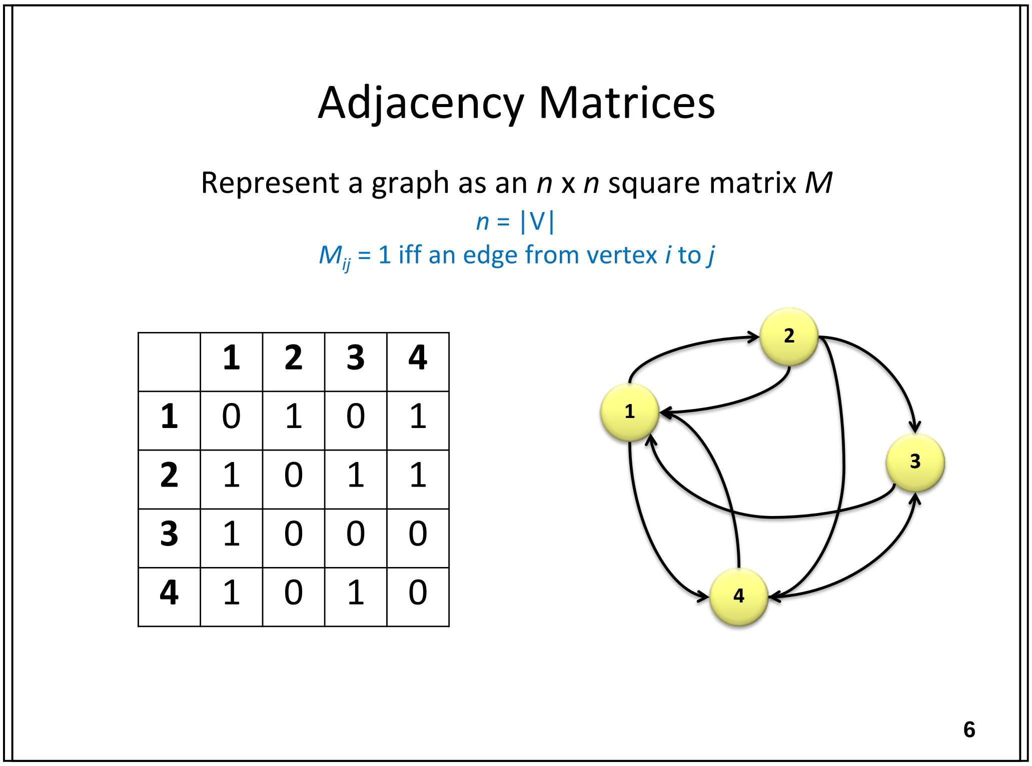
- Social networks
- Hyperlink structure of the web
- Computers on the Internet

We're mostly interested in sparse graphs!



Representing Graphs

- Adjacency matrices
- Adjacency lists
- Edge lists



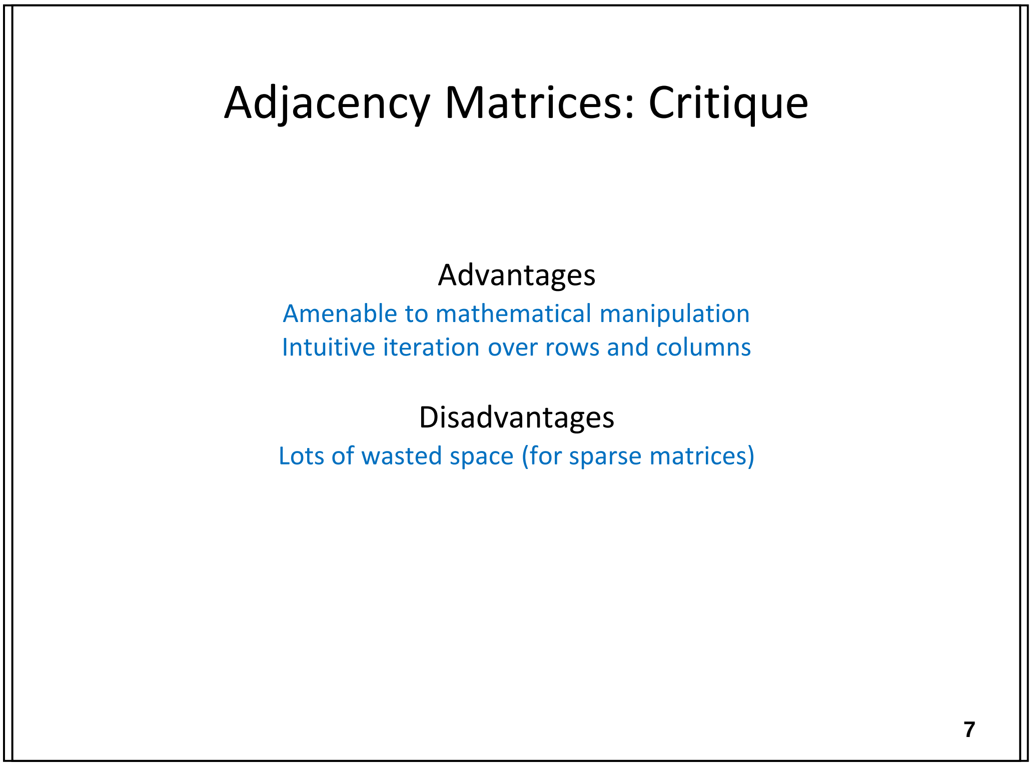
Adjacency Matrices

Represent a graph as an *n* x *n* square matrix *M*

$n = |V|$

$M_{ij} = 1$ iff an edge from vertex *i* to *j*

	1	**2**	**3**	**4**
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0



Adjacency Matrices: Critique

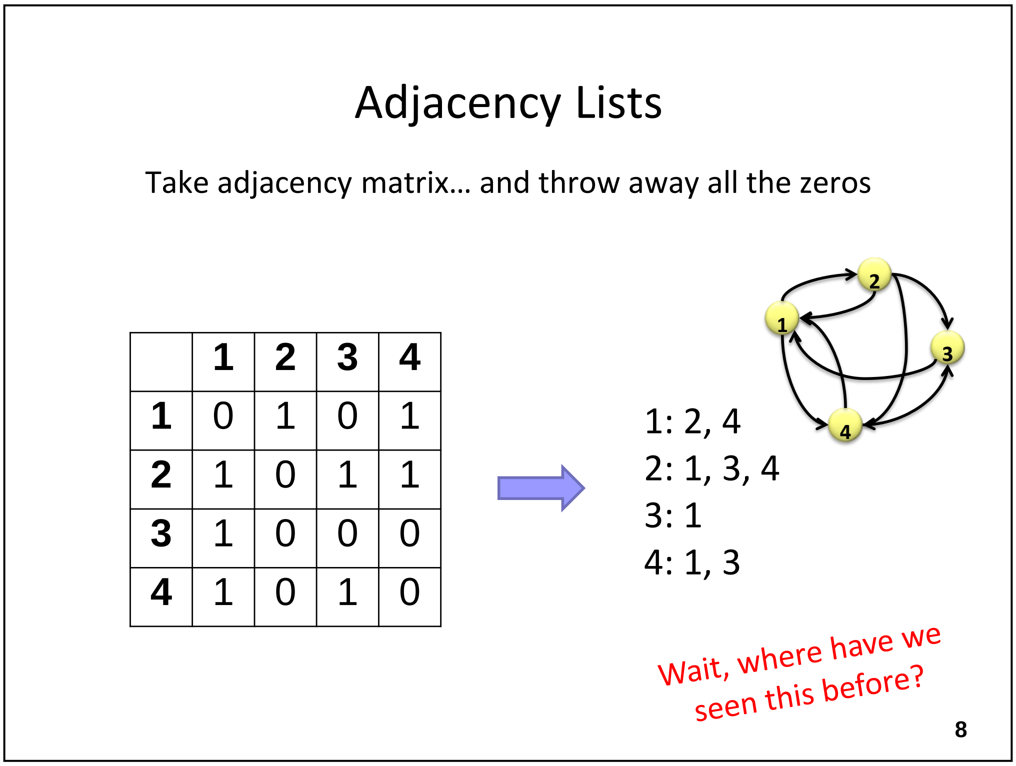
Advantages

Amenable to mathematical manipulation

Intuitive iteration over rows and columns

Disadvantages

Lots of wasted space (for sparse matrices)



Adjacency Lists

Take adjacency matrix… and throw away all the zeros

	1	**2**	**3**	**4**
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0

⇨

1: 2, 4
2: 1, 3, 4
3: 1
4: 1, 3

Wait, where have we seen this before?

We have seen this in posting lists.

Adjacency Lists: Critique

Advantages

Much more compact representation (compress!)

Easy to compute over outlinks

Disadvantages

Difficult to compute over inlinks

Edge Lists

Explicitly enumerate all edges

	1	**2**	**3**	**4**
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0

(1, 2)
(1, 4)
(2, 1)
(2, 3)
(2, 4)
(3, 1)
(4, 1)
(4, 3)

Edge Lists: Critique

Advantages

Easily support edge insertions

Disadvantages

Wastes spaces

Some Graph Problems

Finding shortest paths

Routing Internet traffic and UPS trucks

Finding minimum spanning trees

Telco laying down fiber

Finding max flow

Airline scheduling

Identify “special” nodes and communities

Halting the spread of avian flu

Bipartite matching

match.com

Web ranking

PageRank

What does the web look like?

Analysis of a large webgraph from the common crawl: 3.5 billion pages, 129 billion links

Meusel et al. Graph Structure in the Web — Revisited. WWW 2014.

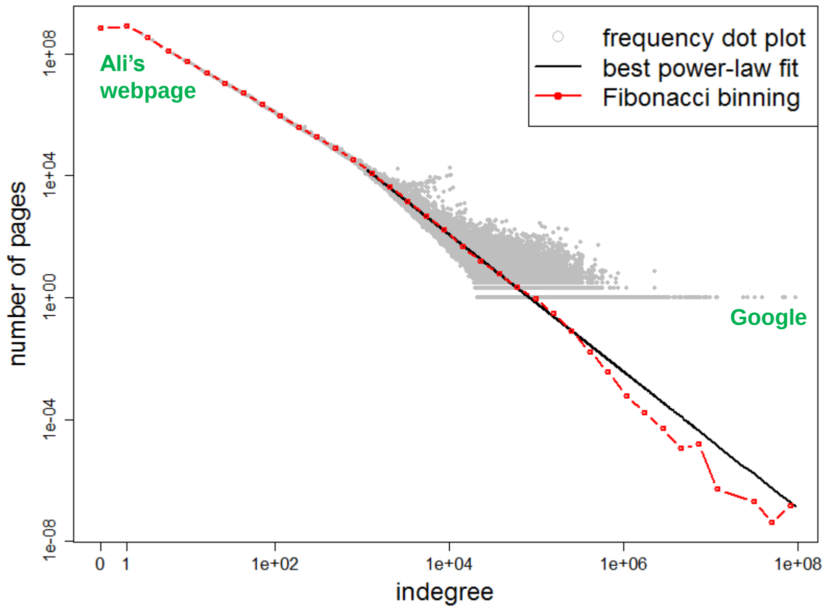
What does the web look like?

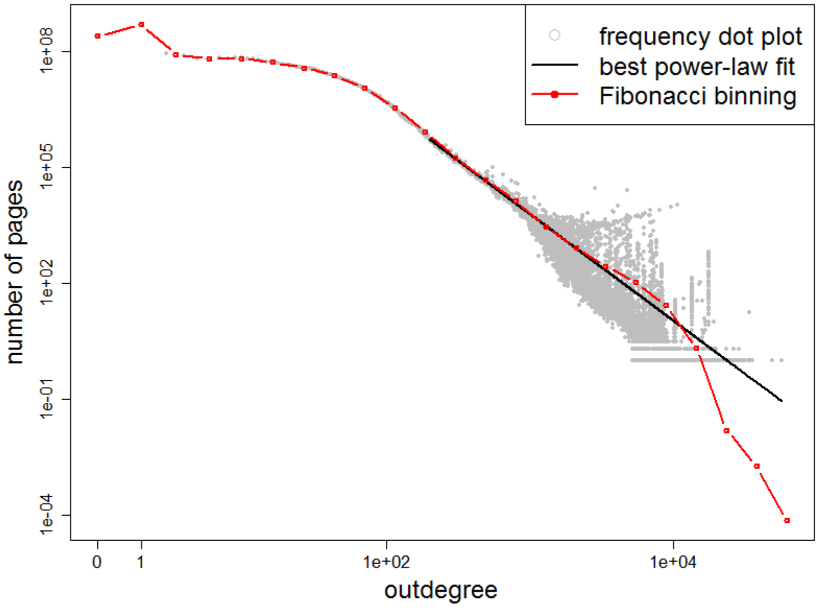
Very roughly, a scale-free network

Fraction of k nodes having k connections:

$$P(k) \sim k^{-\gamma}$$

(i.e., degree distribution follows a power law)


frequency dot plot
best power-law fit
Fibonacci binning
Ali's webpage
Google
number of pages
1e+08
1e+04
1e+00
1e-04
1e-08
indegree
0
1
1e+02
1e+04
1e+06
1e+08



frequency dot plot
best power-law fit
Fibonacci binning
number of pages
1e+08
1e+05
1e+02
1e-01
1e-04
0
1
1e+02
1e+04
outdegree

How do we extract the webgraph?
The webgraph… is big?!

webgraph from the common crawl: 3.5 billion pages, 129 billion links

Meusel et al. Graph Structure in the Web — Revisited. WWW 2014.

58 GB!

Graphs and MapReduce (and Spark)

A large class of graph algorithms involve:

Local computations at each node

Propagating results: “traversing” the graph

Key questions:

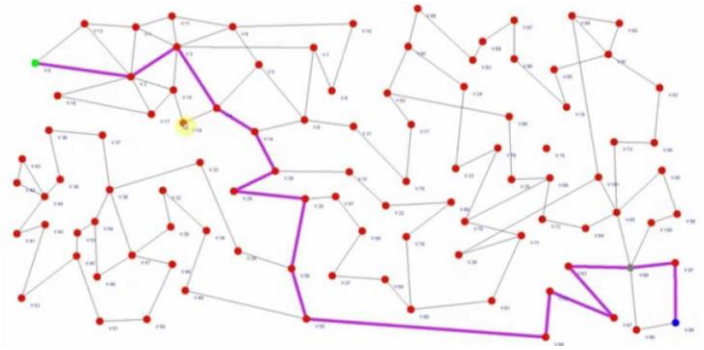
How do you represent graph data in MapReduce (and Spark)?

How do you traverse a graph in MapReduce (and Spark)?

Single-Source Shortest Path

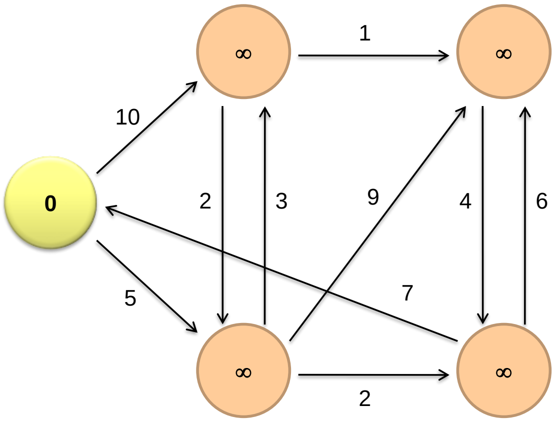
Problem: find shortest path from a
source node to one or more target nodes

Shortest might also mean lowest weight or cost



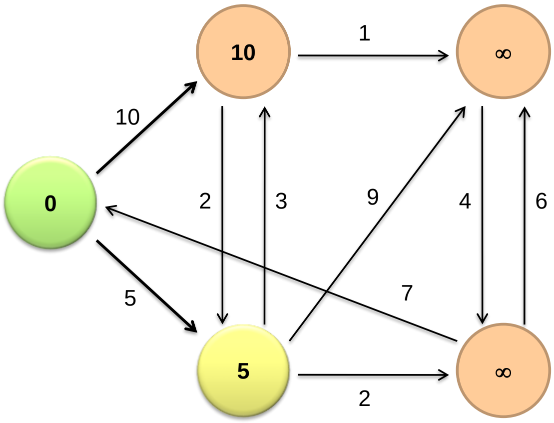
First, a refresher: Dijkstra's Algorithm…

Dijkstra’s Algorithm Example



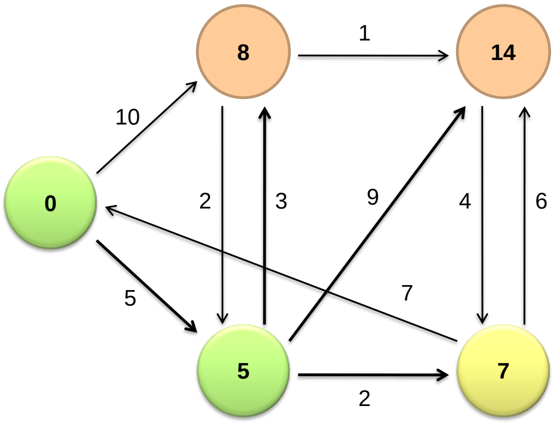
Example from CLR

Dijkstra’s Algorithm Example



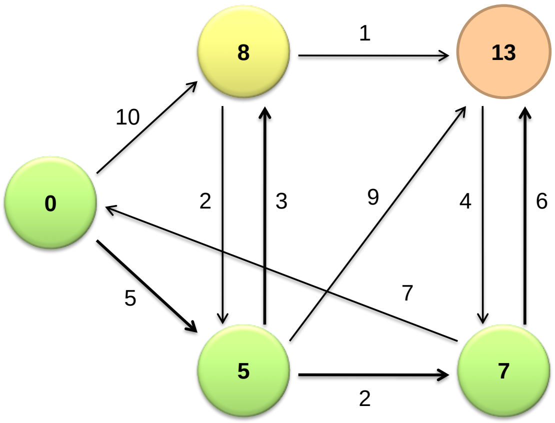
Example from CLR

Dijkstra's Algorithm Example



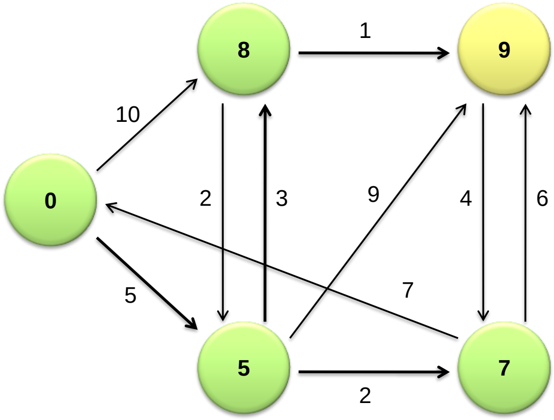
Example from CLR

Dijkstra’s Algorithm Example



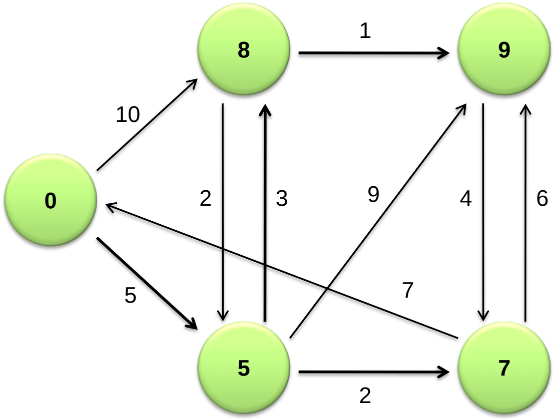
Example from CLR

Dijkstra’s Algorithm Example



Example from CLR

Dijkstra’s Algorithm Example



Example from CLR

"Simplicity is prerequisite for reliability."

Edsger Dijkstra

Single-Source Shortest Path

Problem: find shortest path from a
source node to one or more target nodes

Shortest might also mean lowest weight or cost

Single processor machine: Dijkstra’s Algorithm

MapReduce: parallel breadth-first search (BFS)

Finding the Shortest Path

Consider simple case of equal edge weights

Solution to the problem can be defined inductively:

Define: *b* is reachable from *a* if *b* is on adjacency list of *a*

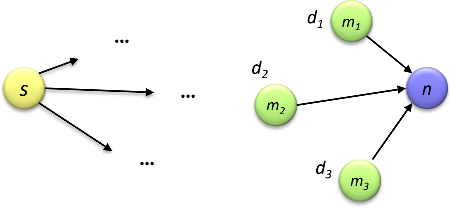
DistanceTo(*s*) = 0

For all nodes *p* reachable from *s*,

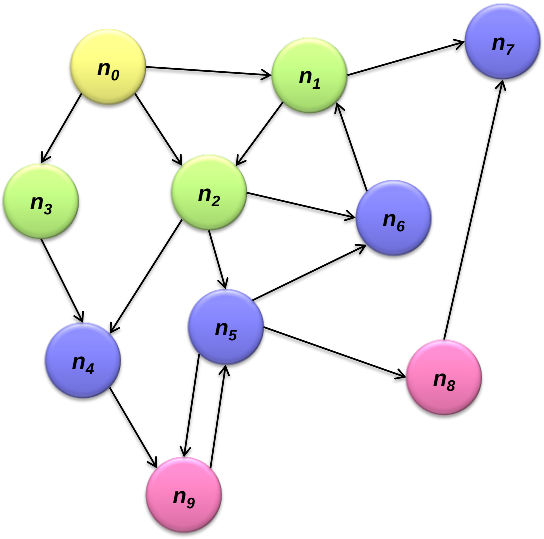
DistanceTo(*p*) = 1

For all nodes *n* reachable from some other set of nodes *M*,

DistanceTo(*n*) = 1 + min(DistanceTo(*m*), $m \in M$)



Visualizing Parallel BFS



From Intuition to Algorithm

Data representation:

Key: node n

Value: d (distance from start), adjacency list

Initialization: for all nodes except for start node, $d = \infty$

Mapper:

$\forall m \in$ adjacency list: emit $(m, d + 1)$

Sort/Shuffle:

Groups distances by reachable nodes

Reducer:

Selects minimum distance path for each reachable node

Additional bookkeeping needed to keep track of actual path

Multiple Iterations Needed

Each MapReduce iteration advances the “frontier” by one hop

Subsequent iterations include more reachable nodes as frontier expands

Multiple iterations are needed to explore entire graph

Preserving graph structure:

Problem: Where did the adjacency list go?

Solution: mapper emits (*n*, adjacency list) as well

Ugh! This is ugly!

BFS Pseudo-Code

distance
adjacencyList

```
class Mapper {
  def map(id: Long, n: Node) = {
    emit(id, n) // emit graph structure
    val d = n.distance
    for (m <- n.adjacencyList) {
      emit(m, d+1)
    }
}

class Reducer {
  def reduce(id: Long, objects: Iterable[Object]) = {
    var min = infinity
    var m = null
    for (d <- objects) {
      if (isNode(d))    m <- d
      else if d < min   min = d
    }
    m.distance = min
    emit(id, m)
  }
}
```

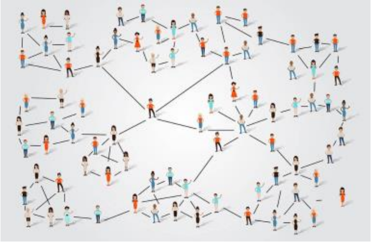
Stopping Criterion

(equal edge weight)

How many iterations are needed in parallel BFS?

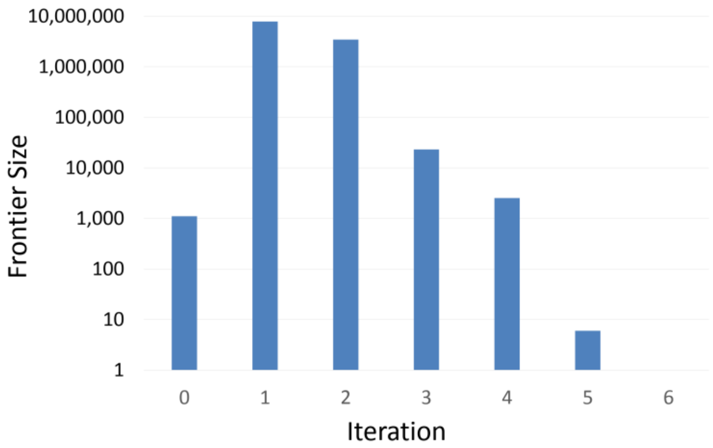
Convince yourself: when a node is first "discovered", we've found the shortest path

What does it have to do with
six degrees of separation?



https://www.csauthors.net/distance

Frontier size during BFS traversal



Last term after this lecture …

Hi professor,

I love your admiration for Ma Long almost as much as I love Ma Long myself … I have some links to Ma Long …

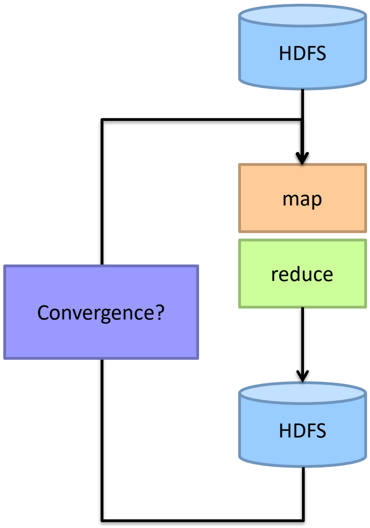
You -> Me -> Wang Chuqin -> Ma Long OR

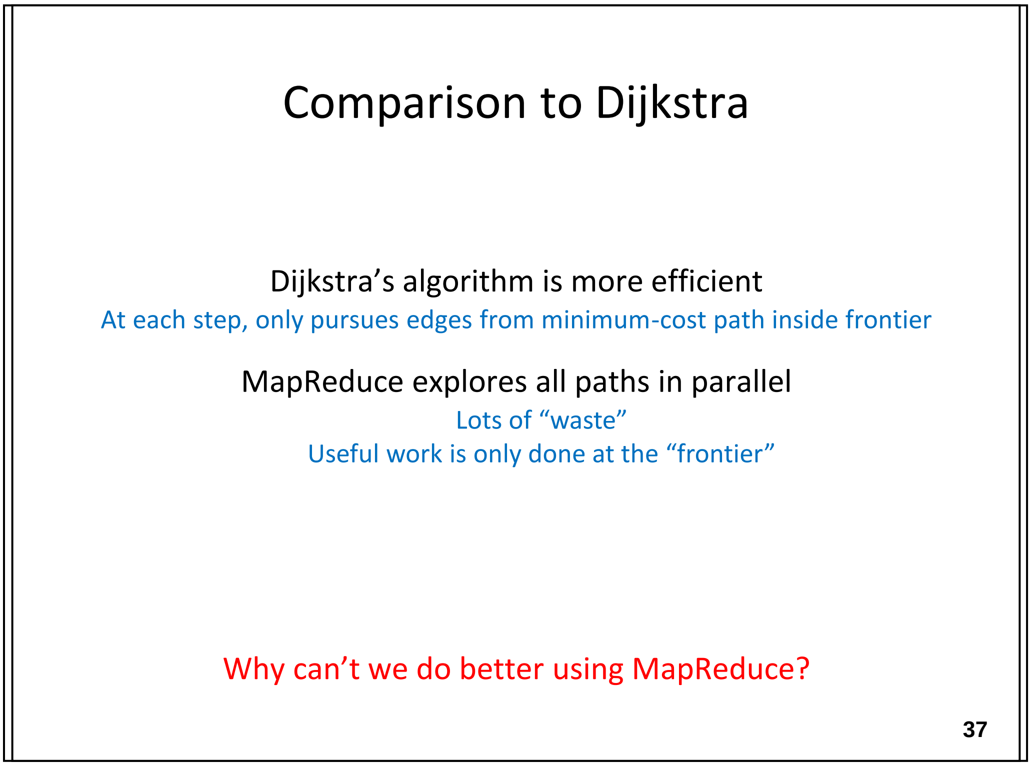
You -> Me -> Kanak Jha -> Ma Long OR

You -> Me -> My Coach -> His Father (former national team coach) -> Ma Long

Turns out, it's a small world indeed.

Implementation Practicalities





Comparison to Dijkstra

Dijkstra’s algorithm is more efficient

At each step, only pursues edges from minimum-cost path inside frontier

MapReduce explores all paths in parallel

Lots of “waste”

Useful work is only done at the “frontier”

Why can’t we do better using MapReduce?

We can’t do better because we cannot keep a global state like Dijkstra does.

Single Source: Weighted Edges

Now add positive weights to the edges

Simple change: add weight w for each edge in adjacency list

Simple change: add weight *w* for each edge in adjacency list

In mapper, emit (m, $d + w_p$) instead of (m, $d + 1$) for each node m

That's it?

Stopping Criterion

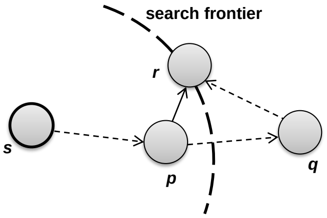
(positive edge weight)

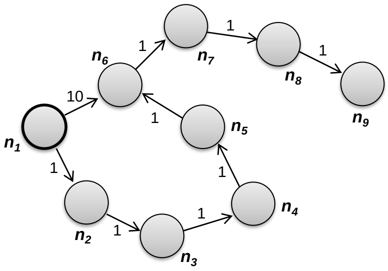
How many iterations are needed in parallel BFS?

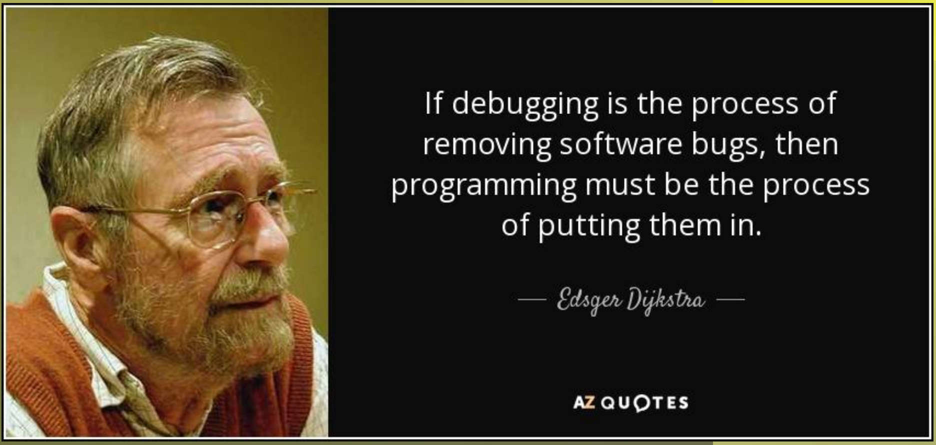
Convince yourself: when a node is first "discovered", we've found the shortest path

Not true!

Additional Complexities







If debugging is the process of removing software bugs, then programming must be the process of putting them in.

— *Edsger Dijkstra* —