# Data-Intensive Distributed Computing

CS 431/631 451/651 (Fall 2021)

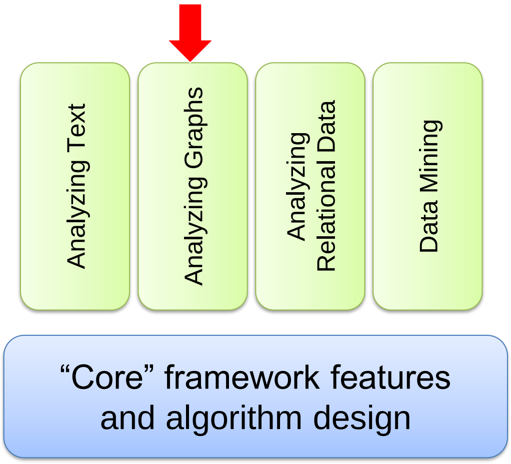
Part 5: Analyzing Graphs (2/2)

Ali Abedi

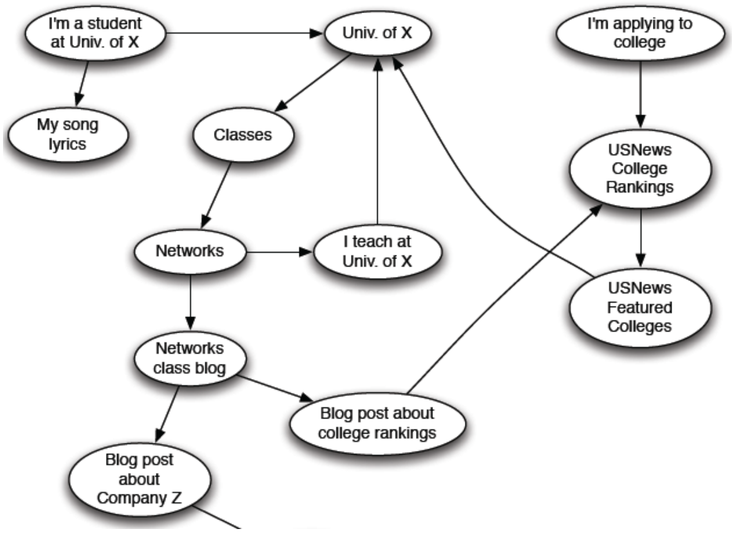
**Thanks to Jure Leskovec, Anand Rajaraman, Jeff Ullman (Stanford University)**

These slides are available at https://www.student.cs.uwaterloo.ca/~cs451/

# Structure of the Course



# Web as a Directed Graph




3

# Who to trust?

**Query: University of Waterloo**

**uwaterloo.ca**


**fakeuw.ca**

University of waterloo University of waterloo University of waterloo University of waterloo University of waterloo University of waterloo University of waterloo University of waterloo

**Ranked retrieval fails!**

# Web Search Challenge

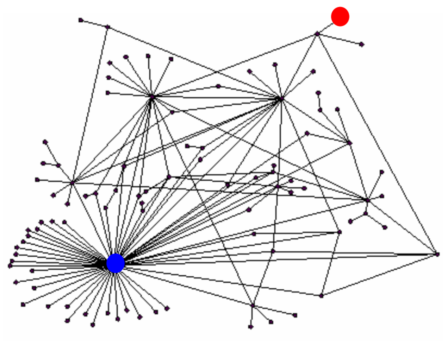
- **Web contains many sources of information**
  **Who to "trust"?**
  - **Trick:** Trustworthy pages may point to each other!

# Ranking Nodes on the Graph

- **All web pages are not equally "important"**

  www.joeschmoe.com vs. www.stanford.edu

- There is large diversity in the web-graph node connectivity.
  **Let's rank the pages by the link structure!**



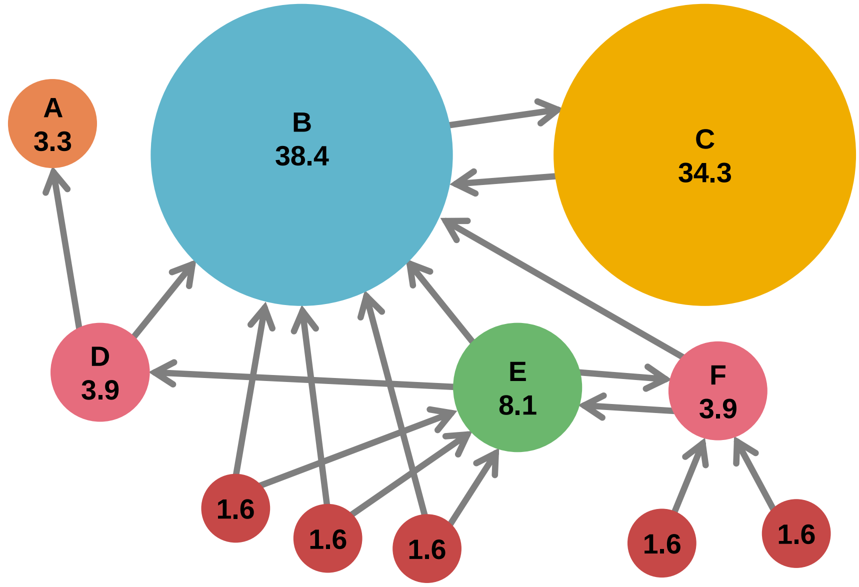
6

# PageRank:
# The "Flow" Formulation

# Links as Votes

- **Idea: Links as votes**
  - **Page is more important if it has more links**
    - In-coming links? Out-going links?
- **Think of in-links as votes:**
  - www.stanford.edu has 23,400 in-links
  - www.joeschmoe.com has 1 in-link
- **Are all in-links equal?**
  - **Links from important pages count more**
  - Recursive question!

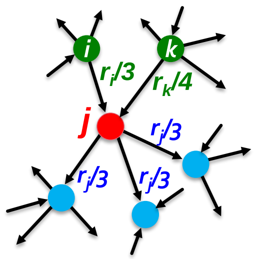
# Example: PageRank Scores



# Simple Recursive Formulation

- Each link's vote is proportional to the **importance** of its source page
- If page ***j*** with importance $r_j$ has ***n*** out-links, each link gets $r_j / n$ votes
- Page ***j***'s own importance is the sum of the votes on its in-links

$r_j = r_i/3 + r_k/4$

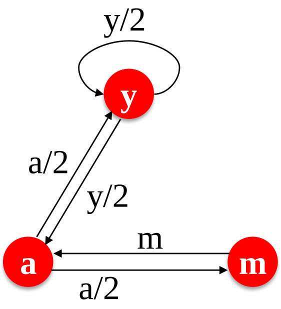


# PageRank: The "Flow" Model

- **Define a "rank" $r_j$ for page $j$**

$$r_j = \sum_{i \to j} \frac{r_i}{\mathrm{d_i}}$$

$d_i$ … out-degree of node $i$




"Flow" equations:

$r_y = r_y/2 + r_a/2$

$r_a = r_y/2 + r_m$

$r_m = r_a/2$

# Solving the Flow Equations

Flow equations:

$r_y = r_y/2 + r_a/2$

$r_a = r_y/2 + r_m$

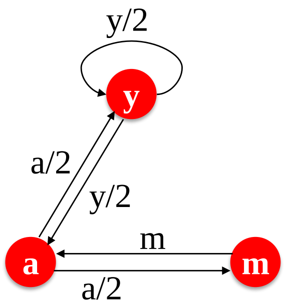
$r_m = r_a/2$

- **3 equations, 3 unknowns, no constants**
  - No unique solution
  - All solutions equivalent modulo the scale factor
- **Additional constraint forces uniqueness:**
  - $\boldsymbol{r_y + r_a + r_m = 1}$
  - **Solution:** $\boldsymbol{r_y = \frac{2}{5},\ r_a = \frac{2}{5},\ r_m = \frac{1}{5}}$
- **Gaussian elimination method works for small examples, but we need a better method for large web-size graphs**
- **We need a new formulation!**

12

# PageRank: Matrix Formulation

- **Stochastic adjacency matrix $M$**
  - Let page $i$ has $d_i$ out-links
  - If $i \rightarrow j$, then $M_{ji} = \frac{1}{d_i}$ else $M_{ji} = 0$
    - $\boldsymbol{M}$ is a **column stochastic matrix**
      - Columns sum to 1



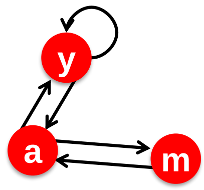

| | y | a | m |
|---|---|---|---|
| **y** | ½ | ½ | 0 |
| **a** | ½ | 0 | 1 |
| **m** | 0 | ½ | 0 |

# PageRank: How to solve?

- **Power Iteration:**
  - Set $r_j = 1/N$
  - **1:** $r'_j = \sum_{i \to j} \frac{r_i}{d_i}$
  - **2:** $r = r'$
  - Goto **1**
- **Example:**

$$\begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix} = \begin{matrix} 1/3 \\ 1/3 \\ 1/3 \end{matrix}$$

Iteration 0, 1, 2, …

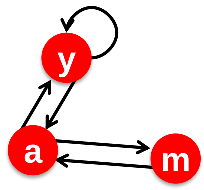



| | y | a | m |
|---|---|---|---|
| y | ½ | ½ | 0 |
| a | ½ | 0 | 1 |
| m | 0 | ½ | 0 |

$r_y = r_y/2 + r_a/2$
$r_a = r_y/2 + r_m$
$r_m = r_a/2$

# PageRank: How to solve?

- **Power Iteration:**
  - Set $r_j = 1/N$
  - **1:** $r'_j = \sum_{i \to j} \frac{r_i}{d_i}$
  - **2:** $r = r'$
  - Goto **1**
- **Example:**




| | y | a | m |
|---|---|---|---|
| y | ½ | ½ | 0 |
| a | ½ | 0 | 1 |
| m | 0 | ½ | 0 |

$r_y = r_y/2 + r_a/2$

$r_a = r_y/2 + r_m$

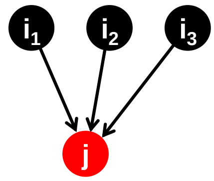
$r_m = r_a/2$

$$\begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix} = \begin{array}{cccccc} 1/3 & 1/3 & 5/12 & 9/24 & & 6/15 \\ 1/3 & 3/6 & 1/3 & 11/24 & \ldots & 6/15 \\ 1/3 & 1/6 & 3/12 & 1/6 & & 3/15 \end{array}$$

Iteration 0, 1, 2, …

# Random Walk Interpretation

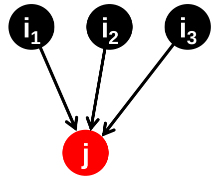
- **Imagine a random web surfer:**
  - At any time $\boldsymbol{t}$, surfer is on some page $\boldsymbol{i}$
  - At time $\boldsymbol{t+1}$, the surfer follows an out-link from $\boldsymbol{i}$ uniformly at random
  - Ends up on some page $\boldsymbol{j}$ linked from $\boldsymbol{i}$
  - Process repeats indefinitely




$$r_j = \sum_{i \to j} \frac{r_i}{\mathrm{d}_{\mathrm{out}}(\mathrm{i})}$$

# Random Walk Interpretation

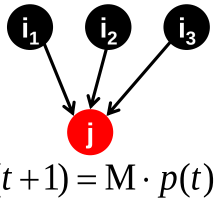
- **Imagine a random web surfer:**
  - At any time $\boldsymbol{t}$, surfer is on some page $\boldsymbol{i}$
  - At time $\boldsymbol{t}+\mathbf{1}$, the surfer follows an out-link from $\boldsymbol{i}$ uniformly at random
  - Ends up on some page $\boldsymbol{j}$ linked from $\boldsymbol{i}$
  - Process repeats indefinitely




$$r_j = \sum_{i \to j} \frac{r_i}{\mathrm{d}_{\mathrm{out}}(\mathrm{i})}$$

- **Let:**
  - $\boldsymbol{p}(\boldsymbol{t})$ … vector whose $\boldsymbol{i}^{\text{th}}$ coordinate is the prob. that the surfer is at page $\boldsymbol{i}$ at time $\boldsymbol{t}$
  - So, $\boldsymbol{p}(\boldsymbol{t})$ is a probability distribution over pages

## The Stationary Distribution




- **Where is the surfer at time $t+1$?**
  - Follows a link uniformly at random

    $$\boldsymbol{p}(\boldsymbol{t}+\mathbf{1}) = \boldsymbol{M} \cdot \boldsymbol{p}(\boldsymbol{t})$$
- Suppose the random walk reaches a state

  $$\boldsymbol{p}(\boldsymbol{t}+\mathbf{1}) = \boldsymbol{M} \cdot \boldsymbol{p}(\boldsymbol{t}) = \boldsymbol{p}(\boldsymbol{t})$$

  then $\boldsymbol{p}(\boldsymbol{t})$ is **stationary distribution** of a random walk

## Existence and Uniqueness

- **A central result from the theory of random walks (a.k.a. Markov processes):**

For graphs that satisfy **certain conditions**, the **stationary distribution is unique** and eventually will be reached no matter what the initial probability distribution at time **t = 0**

# PageRank:
# The Google Formulation

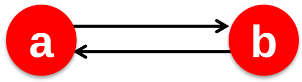
20

# PageRank: Three Questions

$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{\mathrm{d_i}}$$

- Does this converge?
- Does it converge to what we want?
- Are results reasonable?

# Does this converge?



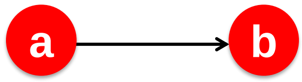

$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{\mathrm{d_i}}$$

- **Example:**

$$\begin{matrix} r_a \\ r_b \end{matrix} = \begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{matrix}$$

Iteration 0, 1, 2, …

# Does it converge to what we want?



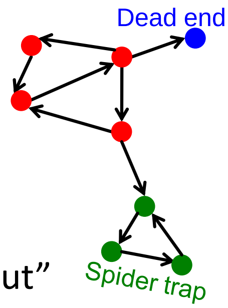

$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{\mathrm{d_i}}$$

- **Example:**

$$\begin{matrix} \mathrm{r_a} \\ \mathrm{r_b} \end{matrix} = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$$

Iteration 0, 1, 2, …

# PageRank: Problems



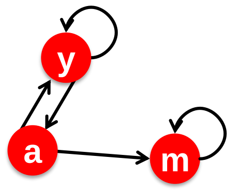

**2 problems:**

- **(1)** Some pages are **dead ends** (have no out-links)
  - Random walk has "nowhere" to go to
  - Such pages cause importance to "leak out"

- **(2) Spider traps:** (all out-links are within the group)
  - Random walker gets "stuck" in a trap
  - And eventually spider traps absorb all importance

# Problem: Spider Traps

- **Power Iteration:**
  - Set $r_j = 1$
  - $r_j = \sum_{i \to j} \frac{r_i}{d_i}$
    - And iterate




m is a spider trap

| | y | a | m |
|---|---|---|---|
| y | ½ | ½ | 0 |
| a | ½ | 0 | 0 |
| m | 0 | ½ | 1 |

$r_y = r_y/2 + r_a/2$
$r_a = r_y/2$
$r_m = r_a/2 + r_m$

- **Example:**

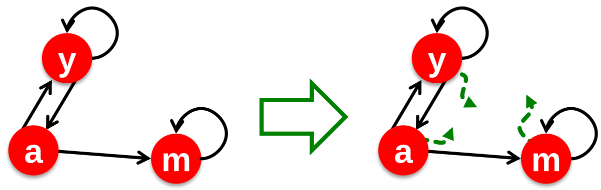
$$\begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix} = \begin{matrix} 1/3 & 2/6 & 3/12 & 5/24 & & 0 \\ 1/3 & 1/6 & 2/12 & 3/24 & \ldots & 0 \\ 1/3 & 3/6 & 7/12 & 16/24 & & 1 \end{matrix}$$

Iteration 0, 1, 2, …

All the PageRank score gets "trapped" in node m.

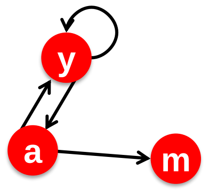
# Solution: Teleports!

- **The Google solution for spider traps: At each time step, the random surfer has two options**
  - With prob. $\beta$, follow a link at random
  - With prob. **1-**$\beta$, jump to some random page
  - Common values for $\beta$ are in the range 0.8 to 0.9
- **Surfer will teleport out of spider trap within a few time steps**



# Problem: Dead Ends

- **Power Iteration:**
  - Set $r_j = 1$
  - $r_j = \sum_{i \to j} \frac{r_i}{d_i}$
    - And iterate




| | y | a | m |
|---|---|---|---|
| y | ½ | ½ | 0 |
| a | ½ | 0 | 0 |
| m | 0 | ½ | 0 |

$r_y = r_y/2 + r_a/2$
$r_a = r_y/2$
$r_m = r_a/2$

- **Example:**

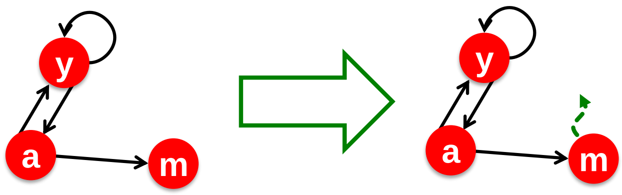
$$\begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix} = \begin{array}{cccccc} 1/3 & 2/6 & 3/12 & 5/24 & & 0 \\ 1/3 & 1/6 & 2/12 & 3/24 & \ldots & 0 \\ 1/3 & 1/6 & 1/12 & 2/24 & & 0 \end{array}$$

Iteration 0, 1, 2, …

Here the PageRank "leaks" out since the matrix is not stochastic.

# Solution: Always Teleport!

- **Teleports:** Follow random teleport links with probability 1.0 from dead-ends
  - Adjust matrix accordingly




| | y | a | m |
|---|---|---|---|
| y | ½ | ½ | 0 |
| a | ½ | 0 | 0 |
| m | 0 | ½ | 0 |

| | y | a | m |
|---|---|---|---|
| y | ½ | ½ | ⅓ |
| a | ½ | 0 | ⅓ |
| m | 0 | ½ | ⅓ |

# Why Teleports Solve the Problem?

**Why are dead-ends and spider traps a problem and why do teleports solve the problem?**

- **Spider-traps** are not a problem, but with traps PageRank scores are **not** what we want
  - **Solution:** Never get stuck in a spider trap by teleporting out of it in a finite number of steps
- **Dead-ends** are a problem
  - The matrix is not column stochastic, so our initial assumptions are not met
  - **Solution:** Make matrix column stochastic by always teleporting when there is nowhere else to go

# Solution: Random Teleports

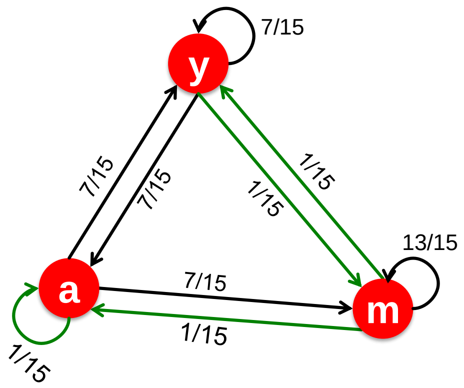
- **Google's solution that does it all:**
  At each step, random surfer has two options:
  - With probability $\beta$, follow a link at random
  - With probability $1\text{-}\beta$, jump to some random page
- **PageRank equation** [Brin-Page, 98]

$$r_j = \sum_{i \to j} \beta \frac{r_i}{d_i} + (1 - \beta) \frac{1}{N}$$

$d_i$ … out-degree of node i

This formulation assumes that $M$ has no dead ends. We can either preprocess matrix $M$ to remove all dead ends or explicitly follow random teleport links with probability 1.0 from dead-ends.

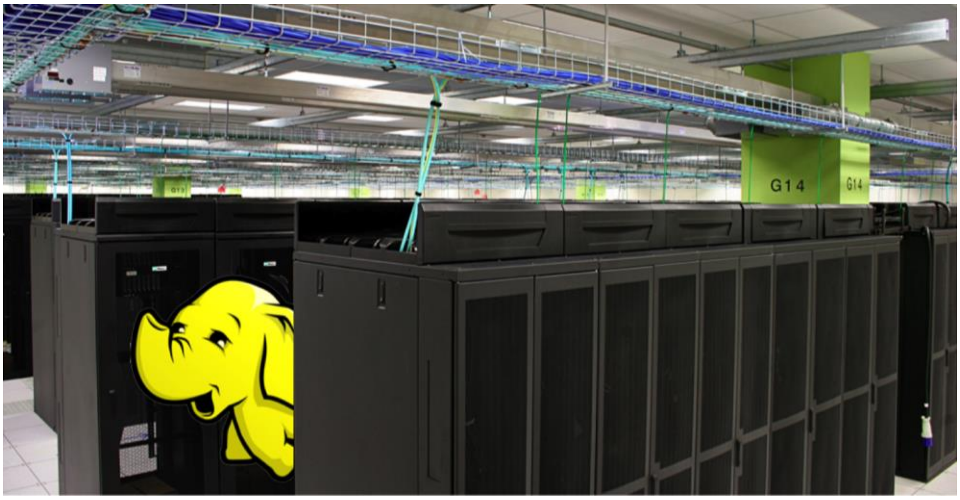
# Random Teleports ($\beta = 0.8$)




$$
0.8 \underbrace{\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}}_{\mathbf{M}} + 0.2 \underbrace{\begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}}_{[1/N]_{NxN}}
$$

$$
\mathbf{A} = \begin{array}{c} y \\ a \\ m \end{array} \begin{bmatrix} 7/15 & 7/15 & 1/15 \\ 7/15 & 1/15 & 1/15 \\ 1/15 & 7/15 & 13/15 \end{bmatrix}
$$

$$
\begin{array}{c} y \\ a \\ m \end{array} = \begin{array}{c} 1/3 \\ 1/3 \\ 1/3 \end{array} \quad \begin{array}{c} 0.33 \\ 0.20 \\ 0.46 \end{array} \quad \begin{array}{c} 0.24 \\ 0.20 \\ 0.52 \end{array} \quad \begin{array}{c} 0.26 \\ 0.18 \\ 0.56 \end{array} \quad \dots \quad \begin{array}{c} 7/33 \\ 5/33 \\ 21/33 \end{array}
$$




# PageRank MapReduce Implementation

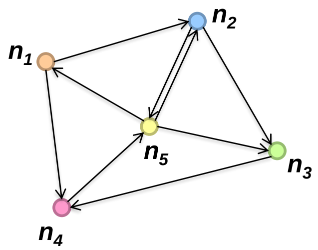
# Simplified PageRank

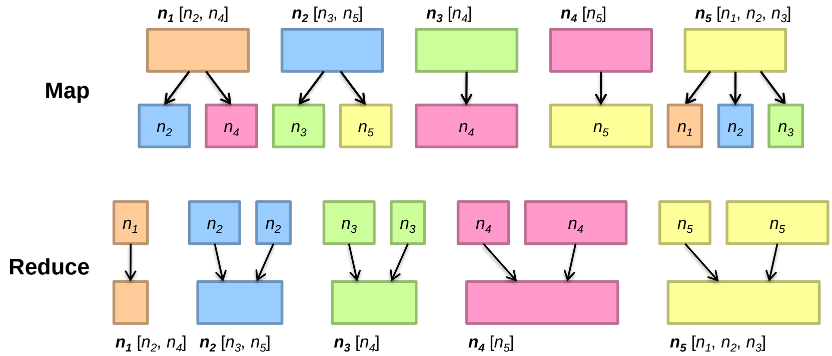
First, tackle the simple case:

No random jump factor

No dangling (dead end) nodes

# PageRank in MapReduce




34

# PageRank Pseudo-Code

```
class Mapper {
  def map(id: Long, n: Node) = {
    emit(id, n)
    p = n.PageRank / n.adjacenyList.length
    for (m <- n.adjacenyList) {
      emit(m, p)
    }
}

class Reducer {
  def reduce(id: Long, objects: Iterable[Object]) = {
    var s = 0
    var n = null
    for (p <- objects) {
      if (isNode(p))
          n = p
      else
          s += p
    }
    n.PageRank = s
    emit(id, n)
  }
}
```

# PageRank vs. BFS

| | PageRank | BFS |
|---|---|---|
| Map | PR/N | d+1 |
| Reduce | sum | min |

A large class of graph algorithms involve:

Local computations at each node

Propagating results: “traversing” the graph

# Complete PageRank

Two additional complexities

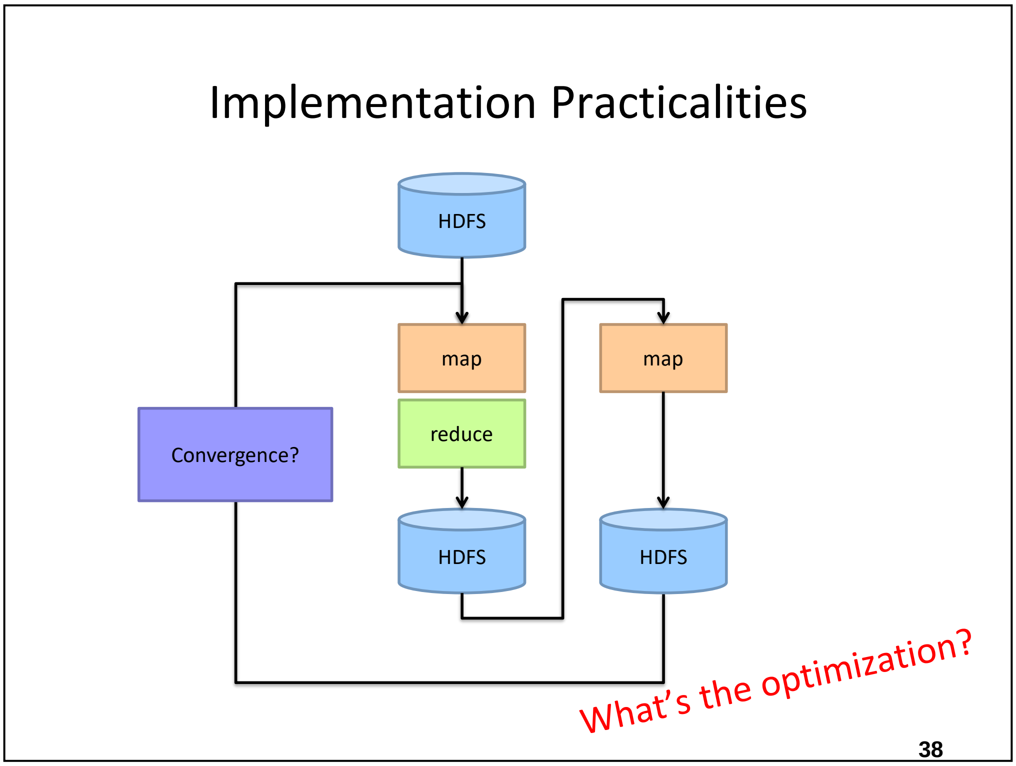
What is the proper treatment of dangling nodes?

How do we factor in the random jump factor?

Solution: second pass to redistribute "missing PageRank mass" and account for random jumps

$$r_j = \sum_{i \to j} \beta \, \frac{r_i}{d_i} + (1 - \beta) \frac{1}{N}$$

One final optimization: fold into a single MR job




Optimization: fold into one MapReduce job

# PageRank Convergence

Alternative convergence criteria

Iterate until PageRank values don't change

Iterate until PageRank rankings don't change

Fixed number of iterations

# Log Probs

PageRank values are *really* small…

Solution?

Product of probabilities = Addition of log probs

Addition of probabilities?

$$a \oplus b = \begin{cases} b + \log(1 + e^{a-b}) & a < b \\ a + \log(1 + e^{b-a}) & a \geq b \end{cases}$$

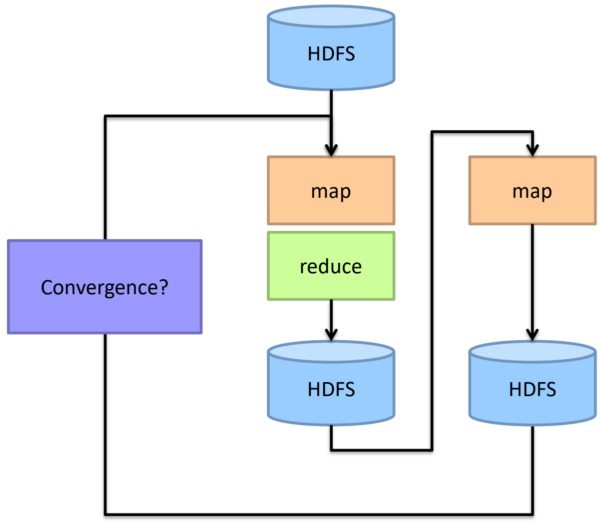
# Beyond PageRank

Variations of PageRank

Weighted edges

Personalized PageRank (A3/A4 ☺)

# Implementation Practicalities




42

# MapReduce Sucks

Java verbosity

Hadoop task startup time

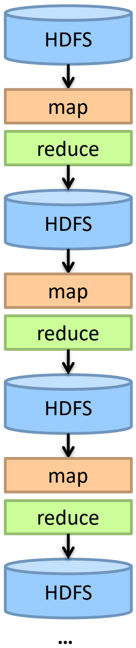
Stragglers

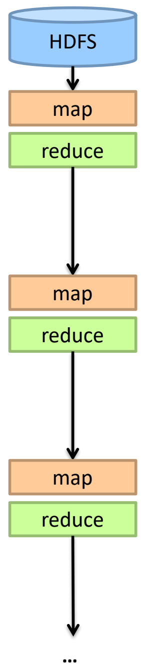
Needless graph shuffling

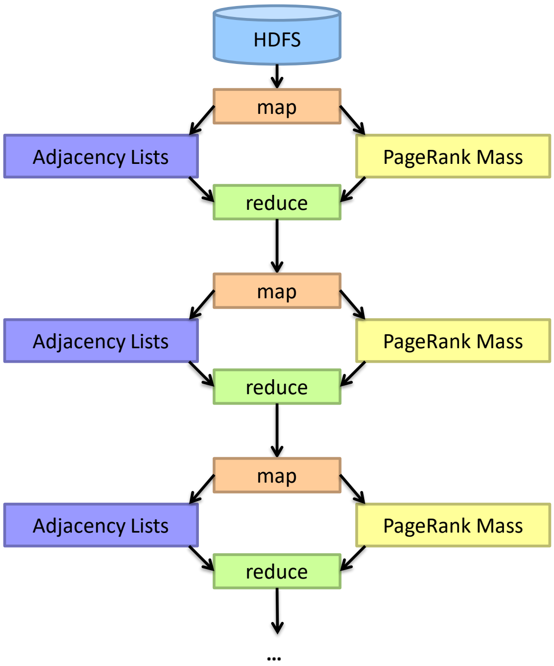
Checkpointing at each iteration

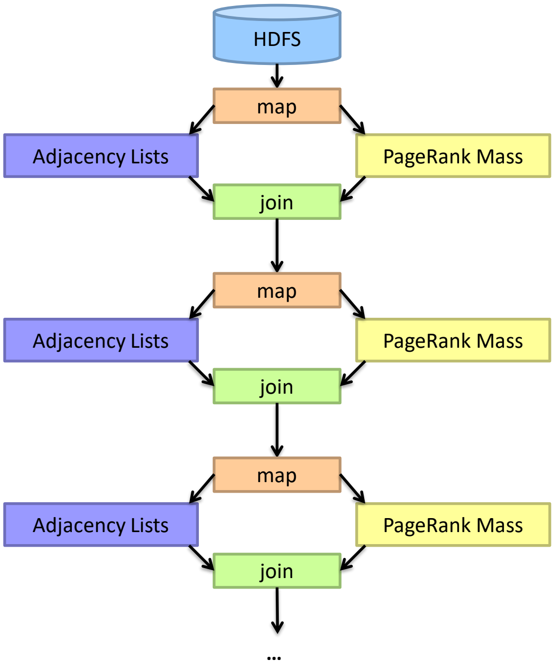
Spark to the rescue?

# Let’s Spark!

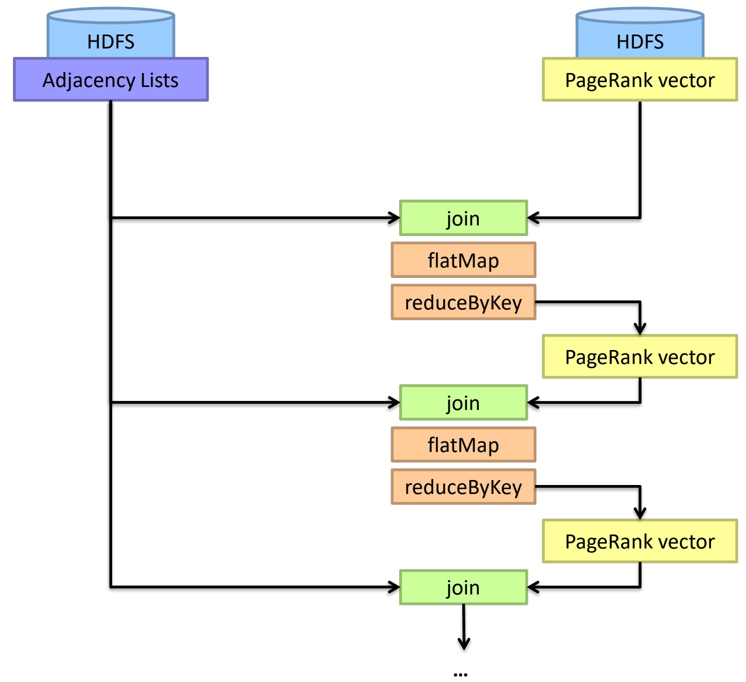



HDFS
map
reduce
map
reduce
map
reduce
...

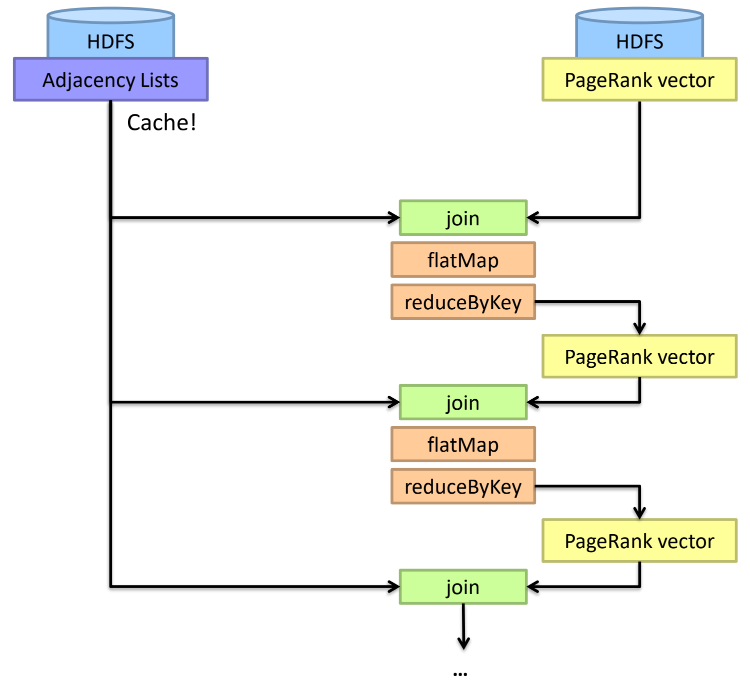

HDFS
map
Adjacency Lists
PageRank Mass
reduce
map
Adjacency Lists
PageRank Mass
reduce
map
Adjacency Lists
PageRank Mass
reduce
…


HDFS
map
Adjacency Lists
PageRank Mass
join
map
Adjacency Lists
PageRank Mass
join
map
Adjacency Lists
PageRank Mass
join
…

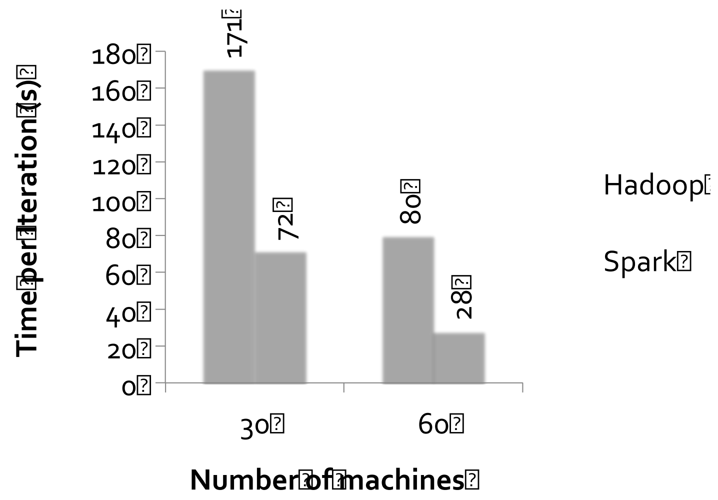
47


HDFS
Adjacency Lists
HDFS
PageRank vector
join
flatMap
reduceByKey
PageRank vector
join
flatMap
reduceByKey
PageRank vector
join
…

48


HDFS
Adjacency Lists
Cache!
HDFS
PageRank vector
join
flatMap
reduceByKey
PageRank vector
join
flatMap
reduceByKey
PageRank vector
join
…

# MapReduce vs. Spark




Source: http://ampcamp.berkeley.edu/wp-content/uploads/2012/06/matei-zaharia-part-2-amp-camp-2012-standalone-programs.pdf