Data-Intensive Distributed Computing

CS 431/631 451/651 (Fall 2021)

Part 7: Data Mining (1/4)

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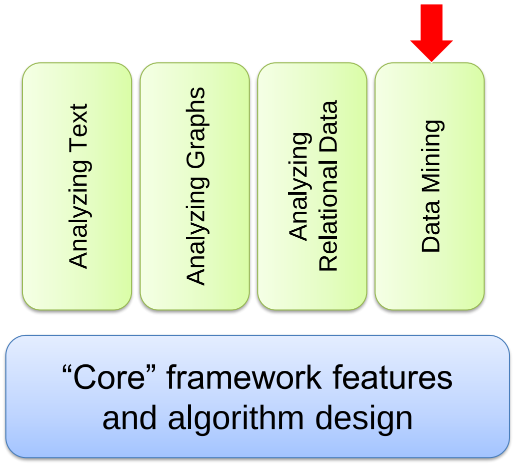
These slides are available at https://www.student.cs.uwaterloo.ca/~cs451

Structure of the Course



Supervised Machine Learning

The generic problem of function induction given sample instances of input and output

Focus today

Classification: output draws from finite discrete labels

Regression: output is a continuous value

This is not meant to be an exhaustive treatment of machine learning!

Classification

Source: Wikipedia (Sorting)

Applications

Spam detection

Sentiment analysis

Content (e.g., topic) classification

Link prediction

Document ranking

Object recognition

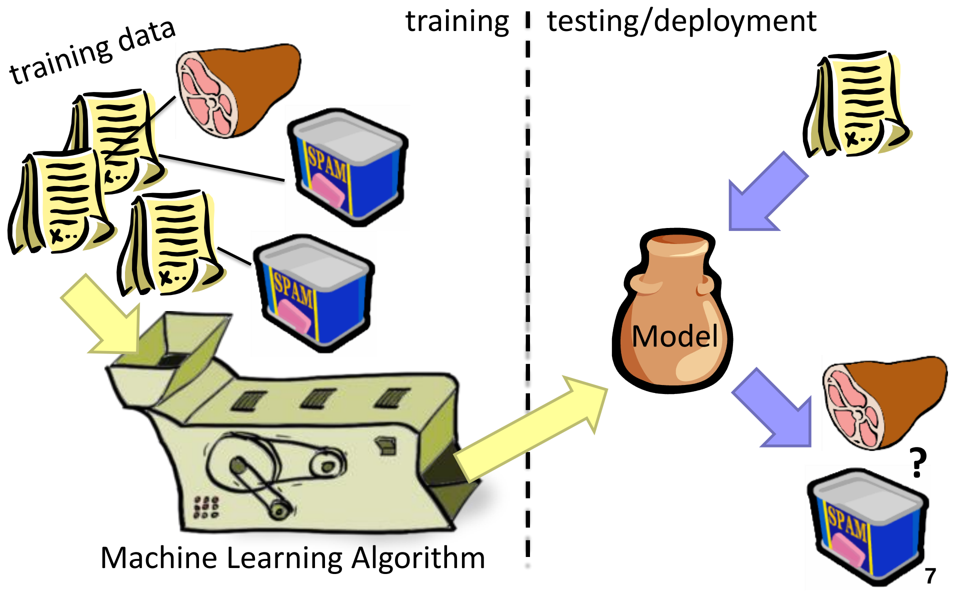
Fraud detection

And much much more!

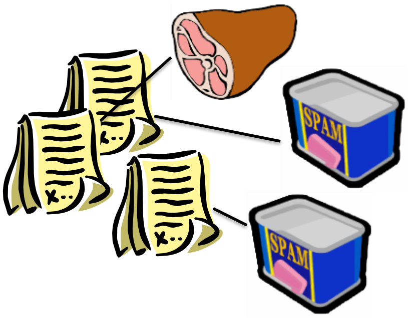
Applications

6

Supervised Machine Learning



Feature Representations



Who comes up with the features?
How?

Objects are represented in terms of features:

- "Dense" features: sender IP, timestamp, # of recipients, length of message, etc.
- "Sparse" features: contains the term "Viagra" in message, contains "URGENT" in subject, etc.

8

Applications

Spam detection

Sentiment analysis

Content (e.g., genre) classification

Link prediction

Document ranking

Object recognition

Fraud detection

And much much more!

Features are highly application-specific!

Components of a ML Solution

Data

Features

Model

Optimization

gradient descent, stochastic gradient descent, L-BFGS, etc.

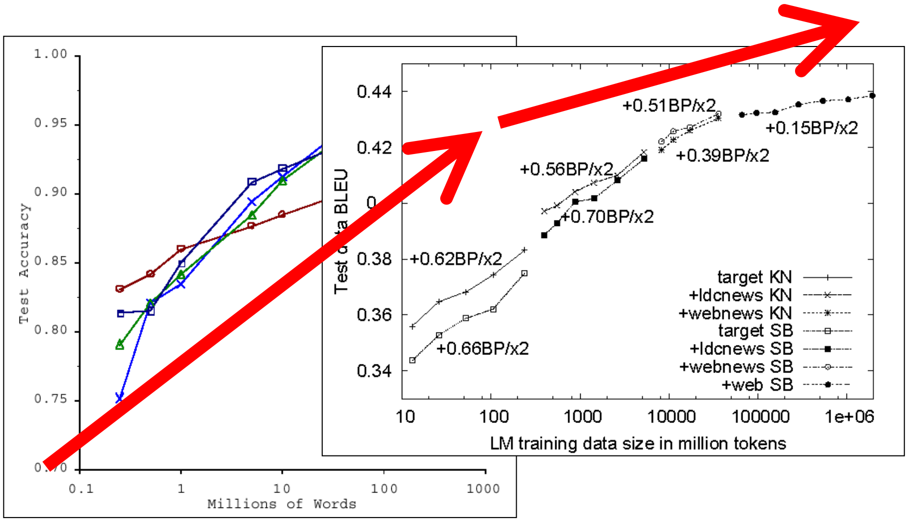
logistic regression, naïve Bayes, SVM, random forests, perceptrons, neural networks, etc.

What "matters" the most?

10

Data matters the most. If we throw a lot of data at almost any algorithm it performs good.

No data like more data!



(Banko and Brill, ACL 2001)
(Brants et al., EMNLP 2007)

11

We have seem these examples before. For example stupid backoff outperforms other algorithms when it’s trained on a lot of data.

Limits of Supervised Classification?

Why is this a big data problem?

Isn't gathering labels a serious bottleneck?

Solutions

Crowdsourcing

Bootstrapping, semi-supervised techniques

Exploiting user behavior logs

Supervised *Binary* Classification

Restrict output label to be *binary*

Yes/No

1/0

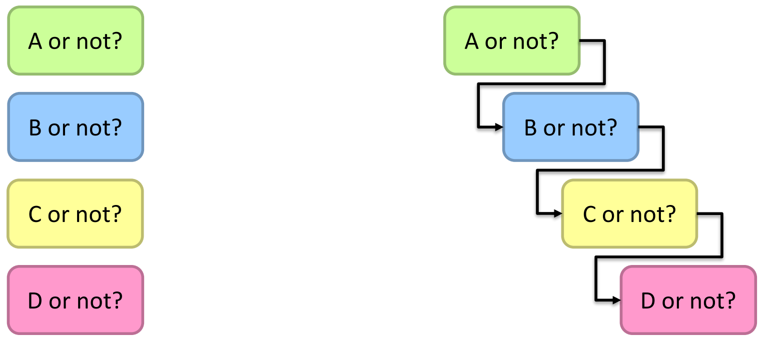
Binary classifiers form primitive
building blocks for multi-class problems…

Binary Classifiers as Building Blocks

Example: four-way classification

One vs. rest classifiers

Classifier cascades



14

The Task

label

Given: $D = \{(\mathrm{x}_i, y_i)\}_i^n$

(sparse) feature vector

$$\mathrm{x}_i = [x_1, x_2, x_3, \ldots, x_d]$$
$$y \in \{0, 1\}$$

Induce: $f : X \rightarrow Y$

Such that loss is minimized

$$\frac{1}{n}\sum_{i=0}^{n} \ell(f(\mathrm{x}_i), y_i)$$

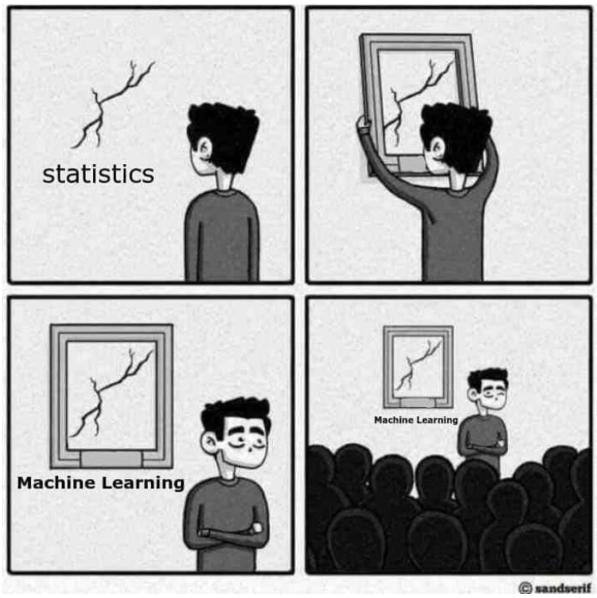
loss function

Typically, we consider functions of a parametric form:

$$\arg\min_{\theta} \frac{1}{n}\sum_{i=0}^{n} \ell(f(x_i;\theta), y_i)$$

model parameters

Key insight: machine learning as an optimization problem!

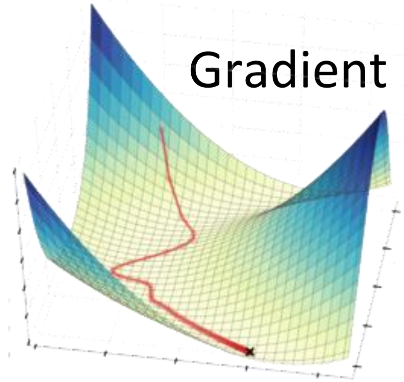


16

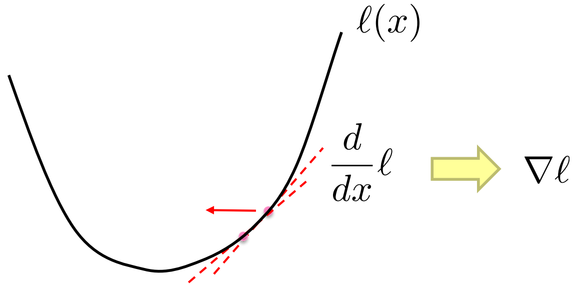
$$\arg\min_{\theta} \frac{1}{n} \sum_{i=0}^{n} \ell(f(x_i;\theta), y_i)$$

Because the closed form solutions generally not possible …

Gradient Descent



Gradient Descent: Preliminaries
Intuition behind the math…



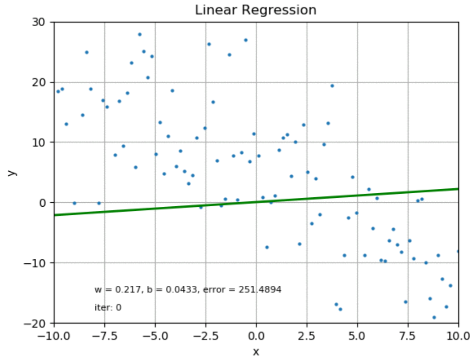
$$\theta^{(t+1)} \leftarrow \theta^{(t)} - \gamma^{(t)} \frac{1}{n} \sum_{i=0}^{n} \nabla \ell(f(\mathrm{x}_i; \theta^{(t)}), y_i)$$

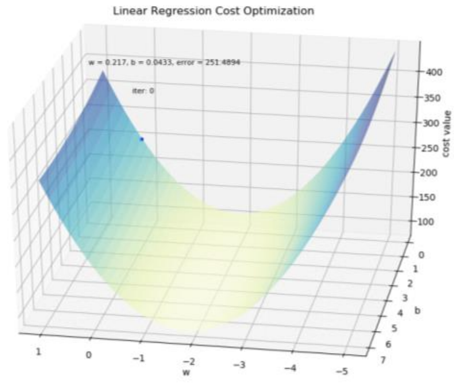
New weights Old weights

Update based on gradient

18

Linear Regression + Gradient Descent





Gradient Descent: Preliminaries

Rewrite:

$$\arg\min_{\theta} \frac{1}{n} \sum_{i=0}^{n} \ell(f(\mathrm{x}_i;\theta), y_i) \quad \Rightarrow \quad \arg\min_{\theta} L(\theta)$$

Compute gradient:

"Points" to fastest increasing "direction"

$$\nabla L(\theta) = \left[\frac{\partial L(\theta)}{\partial w_0}, \frac{\partial L(\theta)}{\partial w_1}, \ldots \frac{\partial L(\theta)}{\partial w_d}\right]$$

So, at any point:

$$\mathrm{b} = \mathrm{a} - \gamma \nabla L(\mathrm{a})$$

$$L(\mathrm{a}) \geq L(\mathrm{b})$$

Gradient Descent: Iterative Update

Start at an arbitrary point, iteratively update:

$$\theta^{(t+1)} \leftarrow \theta^{(t)} - \gamma^{(t)} \nabla L(\theta^{(t)})$$

We have:

$$L(\theta^{(0)}) \geq L(\theta^{(1)}) \geq L(\theta^{(2)}) \ldots$$

Gradient Descent: Iterative Update

Start at an arbitrary point, iteratively update:

$$\theta^{(t+1)} \leftarrow \theta^{(t)} - \gamma^{(t)} \nabla L(\theta^{(t)})$$

We have:

$$L(\theta^{(0)}) \geq L(\theta^{(1)}) \geq L(\theta^{(2)}) \dots$$

Lots of details:

Figuring out the step size

Getting stuck in local minima

Convergence rate

…

Gradient Descent

Repeat until convergence:

$$\theta^{(t+1)} \leftarrow \theta^{(t)} - \gamma^{(t)} \frac{1}{n} \sum_{i=0}^{n} \nabla \ell(f(\mathrm{x}_i; \theta^{(t)}), y_i)$$

Note, sometimes formulated as *ascent* but entirely equivalent

Even More Details…

Gradient descent is a "first order" optimization technique

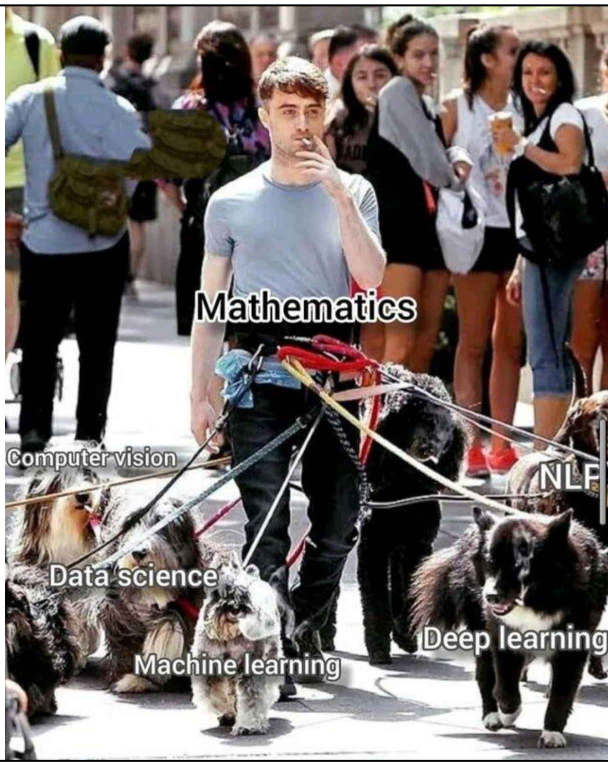
Often, slow convergence

Newton and quasi-Newton methods:

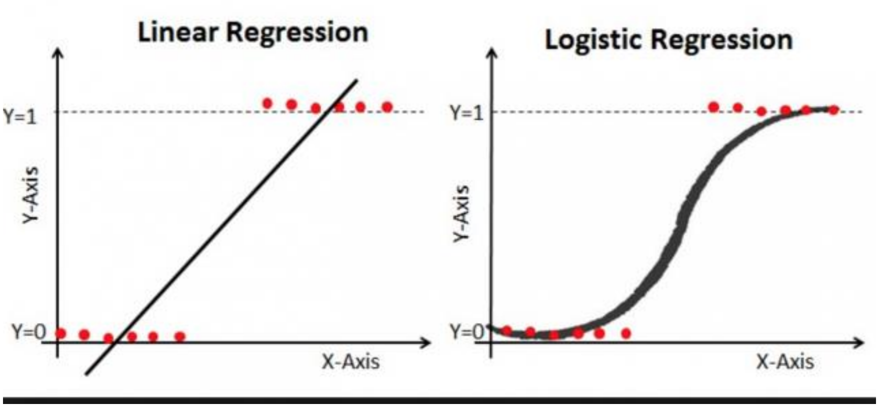
Intuition: Taylor expansion

$$f(x + \Delta x) = f(x) + f'(x)\Delta x + \frac{1}{2}f''(x)\Delta x^2$$

Requires the Hessian (square matrix of second order partial derivatives): impractical to fully compute


Mathematics
Computer vision
NLP
Data science
Deep learning
Machine learning

Logistic Regression: Preliminaries



Logistic Regression: Preliminaries

Given: $D = \{(\mathrm{x}_i, y_i)\}_i^n$

$$\mathrm{x}_i = [x_1, x_2, x_3, \ldots, x_d]$$

$$y \in \{0, 1\}$$

Define: $f(\mathrm{x}; \mathrm{w}) : \mathbb{R}^d \to \{0, 1\}$

$$f(\mathrm{x}; \mathrm{w}) = \begin{cases} 1 \text{ if } \mathrm{w} \cdot \mathrm{x} \geq t \\ 0 \text{ if } \mathrm{w} \cdot \mathrm{x} < t \end{cases}$$

Interpretation: $\ln \left[\frac{\Pr(y = 1|\mathrm{x})}{\Pr(y = 0|\mathrm{x})} \right] = \mathrm{w} \cdot \mathrm{x}$

$$\ln \left[\frac{\Pr(y = 1|\mathrm{x})}{1 - \Pr(y = 1|\mathrm{x})} \right] = \mathrm{w} \cdot \mathrm{x}$$

27

Relation to the Logistic Function

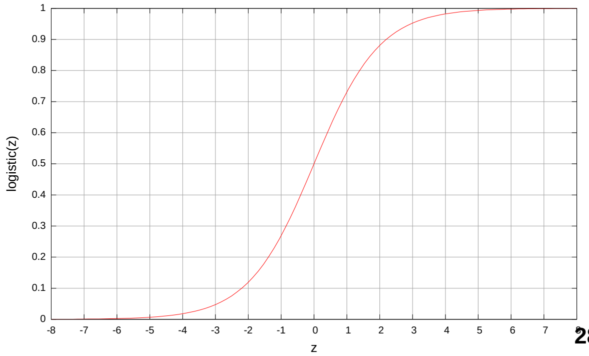
After some algebra:

$$\Pr\left(y=1|x\right)=\frac{e^{\mathrm{w}\cdot\mathrm{x}}}{1+e^{\mathrm{w}\cdot\mathrm{x}}}$$

$$\Pr\left(y=0|x\right)=\frac{1}{1+e^{\mathrm{w}\cdot\mathrm{x}}}$$

The logistic function:

$$f(z)=\frac{e^z}{e^z+1}$$



28

Training an LR Classifier

Maximize the conditional likelihood: $\arg\max_{\mathrm{w}} \prod_{i=1}^{n} \Pr(y_i|\mathrm{x}_i, \mathrm{w})$

Define the objective in terms of conditional *log* likelihood: $L(\mathrm{w}) = \sum_{i=1}^{n} \ln \Pr(y_i|\mathrm{x}_i, \mathrm{w})$

We know: $y \in \{0, 1\}$

So: $Pr(y|\mathrm{x}, \mathrm{w}) = Pr(y = 1|\mathrm{x}, \mathrm{w})^y Pr(y = 0|\mathrm{x}, \mathrm{w})^{(1-y)}$

Substituting:

$$L(\mathrm{w}) = \sum_{i=1}^{n} \Big(y_i \ln \Pr(y_i = 1|\mathrm{x}_i, \mathrm{w}) + (1 - y_i) \ln \Pr(y_i = 0|\mathrm{x}_i, \mathrm{w}) \Big)$$

29

LR Classifier Update Rule

Take the derivative:

$$L(\mathrm{w}) = \sum_{i=1}^{n} \Big(y_i \ln \Pr(y_i = 1|\mathrm{x}_i, \mathrm{w}) + (1 - y_i) \ln \Pr(y_i = 0|\mathrm{x}_i, \mathrm{w}) \Big)$$

$$\frac{\partial}{\partial \mathrm{w}} L(\mathrm{w}) = \sum_{i=0}^{n} \mathrm{x}_i \Big(y_i - \Pr(y_i = 1|\mathrm{x}_i, \mathrm{w}) \Big)$$

General form of update rule:

$$\mathrm{w}^{(t+1)} \leftarrow \mathrm{w}^{(t)} + \gamma^{(t)} \nabla_{\mathrm{w}} L(\mathrm{w}^{(t)})$$

$$\nabla L(\mathrm{w}) = \left[\frac{\partial L(\mathrm{w})}{\partial w_0}, \frac{\partial L(\mathrm{w})}{\partial w_1}, \ldots \frac{\partial L(\mathrm{w})}{\partial w_d} \right]$$

Final update rule:

$$\mathrm{w}_i^{(t+1)} \leftarrow \mathrm{w}_i^{(t)} + \gamma^{(t)} \sum_{j=0}^{n} x_{j,i} \Big(y_j - \Pr(y_j = 1|\mathrm{x}_j, \mathrm{w}^{(t)}) \Big)$$

Lots more details…

Regularization
Different loss functions
…

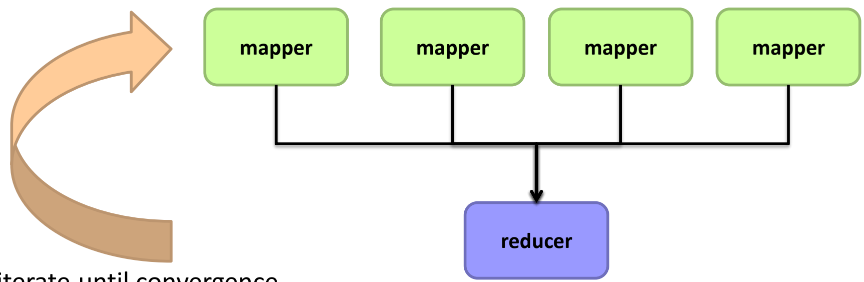
Want more details?
Take a real machine-learning course!

$$\mathrm{w}_i^{(t+1)} \leftarrow \mathrm{w}_i^{(t)} + \gamma^{(t)} \sum_{j=0}^{n} x_{j,i} \Big(y_j - \Pr(y_j = 1 | \mathrm{x}_j, \mathrm{w}^{(t)}) \Big)$$

MapReduce Implementation

$$\underbrace{\theta^{(t+1)} \leftarrow \theta^{(t)} - \gamma^{(t)} \frac{1}{n} \underbrace{\sum_{i=0}^{n} \nabla \ell(f(\mathrm{x}_i; \theta^{(t)}), y_i)}_{\text{mappers}}}_{\text{single reducer}}$$

compute partial gradient



Shortcomings

Hadoop is bad at iterative algorithms

High job startup costs

Awkward to retain state across iterations

High sensitivity to skew

Iteration speed bounded by slowest task

Potentially poor cluster utilization

Must shuffle all data to a single reducer

Some possible tradeoffs

Number of iterations vs. complexity of computation per iteration

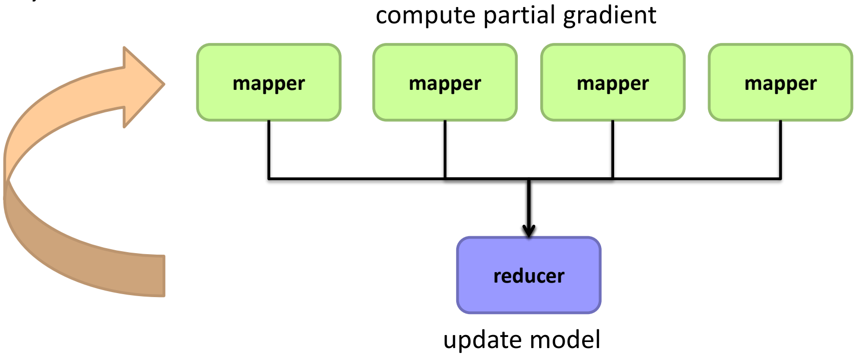
E.g., L-BFGS: faster convergence, but more to compute

Spark Implementation

```
val points = spark.textFile(...).map(parsePoint).persist()

var w = // random initial vector
for (i <- 1 to ITERATIONS) {
  val gradient = points.map{ p =>
    p.x * (1/(1+exp(-p.y*(w dot p.x))-1)*p.y
  }.reduce((a,b) => a+b)
  w -= gradient
}
```

What's the difference?



35