15. Define \( \text{max}(L) = \{ x \in L : x \text{ is not a proper prefix of any } y \in L \} \). Find an expression for \( \text{max}(L) \) in terms of operations like complement, concatenation, etc. Conclude that if \( L \) is regular, so is \( \text{max}(L) \).

Let \( \Sigma \) be the underlying alphabet. We start by observing that the set \( \text{Pref}(L) := \{ z : z \text{ is a proper prefix of some } y \in L \} \) can be compactly written as \( \text{Pref}(L) = L/\Sigma^+ \) since

\[
z \in \text{Pref}(L) \iff \exists z' \in \Sigma^+ \text{ such that } zz' \in L \\
\iff z \in L/\Sigma^+, \text{ by definition of quotient}
\]

Then, by definition of \( \text{max}(L) \) we have \( \text{max}(L) = \overline{\text{Pref}(L)} \cap L = (L/\Sigma^+) \cap L. \)

Finally, \( \text{max}(L) \) is regular by theorem from class (aka \( L/\Sigma^+ \) is regular because regular language quotient anything is regular) and closure properties of regular languages.

![Diagram](image)

Figure 1: \( \text{Pref}(L) = L/\Sigma^+ \)