18. Is the following language regular? \( \{ xwx^R : x, w \in \{0,1\}^+ \} \)

Let \( L = \{ xwx^R : x, w \in \{0,1\}^+ \}, \Sigma = \{0,1\} \). We will show that \( L \) is regular by providing a regular expression \( R \) and showing \( L(R) = L \).

\[
R = (0\{0,1\}^+0) \cup (1\{0,1\}^+1)
\]

We show that \( \forall y \in \Sigma^*, y \in L \iff y \in L(R) \)

( \( \implies \) ): Consider any string \( y \in L \). It obviously has the form \( y = xwx^R \) by definition for some \( x, w \in \Sigma^+ \). Let the first letter of \( x \) be \( a \). Since \( y = xwx^R \), it is clear that \( y \) starts and ends with \( a \) and thus can be rewritten as

\[
y = aw'a, a \in \Sigma, w' \in \Sigma^+
\]

Since \( a \in \Sigma, w' \in \Sigma^+ \), it is clear that \( y \in L(R) \).

( \( \impliedby \) ): Consider any string \( y \in L(R) \).

**Case 1:** Suppose \( y \in L(0\{0,1\}^+0) \). So \( y = 0w0, w \in \Sigma^+ \). Let \( x = 0 = x^R \), and we can clearly see \( y \in L \).

**Case 2:** Nearly identical to case 1