All solutions to the equation $xy^r = yx$ have the form $y = uv, x = (uv)^e u$, where $u, v$ are palindromes and $e \geq 0$. We will prove this by induction on $|xy|$.

The base case is $|x| = |y| = 0$, which implies $x = y = \epsilon$. The statement is true as we can let $u = v = \epsilon$ (which is a palindrome) and let $e = 0$.

Assume that the statement holds for $|xy| \leq k$, and we will show that it is true for $|xy| = k + 1$. Suppose we have $x, y$ such that $xy^r = yx$ and $|xy| = k + 1$.

Case 1: $|y| \geq |x|$. Then $y = xt$ for some $t$. We then have that $xy^r = x t^r x^r = xt x = yx$, which implies $t^r x^r = tx$ after stripping off the leading $x$. Thus, $t^r = t$ and $x^r = x$, so $t$ and $x$ are palindromes. Let $u = x, v = t, e = 0$. Since $x = u = (uv)^0 u$ and $y = xt = uv$, the statement holds.

Case 2: $|y| < |x|$. Then $x = yt$ for some non-empty $t$. Then, $xy^r = y t y^r = y y t = y x$, which gives $ty^r = y t$ after stripping off the leading $y$. Since $|ty| = |x| < |xy|$, we can use the inductive hypothesis, which states that there are palindromes $u, v$ and $e \geq 0$ such that $t = (uv)^e u$ and $y = uv$. Then, since $x = yt = (uv)(uv)^e u = (uv)^{e+1} u$, the statement holds here as well by using $u, v$ and $e + 1$. 

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