Theorem 1. Let $L$ be regular language over $\Delta$. Let $s : \Sigma^* \to 2^{\Delta^*}$ be a substitution that maps each $a \in \Sigma$ to some language $L_a$ that is not necessarily regular. Define
$$s^{-1}(L) = \{x : s(x) \cap L \neq \emptyset\}$$
Then $s^{-1}(L)$ is regular.

Proof. Let $M = (Q, \Delta, \delta, q_0, F)$ be a DFA for $L$.
We will construct an NFA for $s^{-1}(L)$. Define $\delta' : Q \times \Sigma \to 2^Q$ by
$$\delta'(q, a) = \{q' \in Q : \exists w \in s(a) \ q' = \delta(q, w)\}$$
Then let $M'$ be the NFA $(Q, \Sigma, \delta', q_0, F)$.
Since $s$ is a substitution, $s(ab) = s(a)s(b)$, so it follows that for all $x \in \Sigma^*$,
$$\delta'(q, x) = \{q' \in Q : \exists w \in s(x) \ q' = \delta(q, w)\}$$
Then for all $x \in \Sigma^*$, we have
$$x \in L(M') \iff \delta'(q_0, x) \cap F \neq \emptyset$$
$$\iff \exists q \in \delta'(q_0, x) \ q \in F$$
$$\iff \exists w \in s(x) \ \delta(q_0, w) \in F \neq \emptyset$$
$$\iff \exists w \in s(x) \ w \in L(M)$$
$$\iff s(x) \cap L \neq \emptyset$$
$$\iff x \in s^{-1}(L)$$
Therefore $s^{-1}(L) = L(M')$, so it is regular. \qed