**Statement**

Let $\Sigma = \{a\}$ and let $L = \{a^{2^n}: n \geq 0\}$.

Then, every string in $\Sigma^*$ is in a unique equivalence class for the Myhill-Nerode equivalence relation.

**Proof**

Suppose for a contradiction that two distinct strings $x, y \in \Sigma^*$ are in the same equivalence class.

Take without loss of generality that $|x| < |y|$.

Note that $x = a^r$ for some $r \geq 0$.

We remark that there must exist some power $2^k$ of two such that $2^k \geq r$.

Let $m = 2^k - r \geq 0$, so then $x = a^{2^k-m}$.

We then find that $xa^m = a^{2^k-m}a^m = a^{2^k} \in L$.

Since $x \sim y$, we then have that $ya^m \in L$.

So, then $y = a^{2^\ell-m}$ for some $\ell \geq 0$.

Moreover, since $|x| < |y|$, we have $2^k - m < 2^\ell - m$, so $k < \ell$.

Consider now $xa^{2^k+m}$.

We find that $xa^{2^k+m} = a^{2^k-m}a^{2^k+m} = a^{2^{k+1}}$, so then $xa^{2^k+m} \in L$.

From $x \sim y$, we then get that $ya^{2^k+m} \in L$.

So, $ya^{2^k+m} = a^{2^\ell-m}a^{2^k+m} = a^{2^k+2^\ell} \in L$.

Since $0 \leq k < \ell$, we have that $0 < 2^k < 2^\ell$, so $2^\ell < 2^k + 2^\ell < 2^\ell + 2^\ell = 2^{\ell+1}$.

Hence, $2^k + 2^\ell$ cannot be a power of two, giving $a^{2^k+2^\ell} \not\in L$ and yielding a contradiction.

Thus, no two distinct strings $x, y \in \Sigma^*$ are in the same equivalence class.