Here’s how these problem-solving sessions work.

Start by working on the problem assigned to your group. If you find a solution, let me know and sketch it for me. If it’s correct, you then present it. Then write up your solution within one week, in a format I can post for the rest of the class, and send it to me. (A scanned handwritten version is fine, provided it is very neatly written.) If you do all that, you fulfill your 10% course mark allocated to these sessions.

If you quickly succeed in solving a problem (some of them are very easy!), try one of the extra problems at the end.

New problems

70. For an even-length string \( x = a_1a_2 \cdots a_{2n} \), define \( \text{switch}(x) = a_{n+1}a_{n+2} \cdots a_{2n}a_1a_2 \cdots a_n \), an operation that switches the first and last halves of \( x \). Thus, for example, \( \text{switch(\text{casebook})} = \text{bookcase} \). Extend to languages in the obvious way:

\[
\text{switch}(L) = \{ \text{switch}(x) : |x| \text{ even and } x \in L \}.
\]

Show that if \( L \) is regular, then \( \text{switch}(L) \) is a CFL, but not necessarily regular. Show that if \( L \) is context-free, then \( \text{switch}(L) \) need not be a CFL.

72. A run in a string is a maximal block of consecutive identical symbols. Thus, for example, 111223333 has three runs, of length 3, 2, 4. Define \( r(x) \) to be the number of runs of \( x \). Consider

\[
L = \{ x \in \Sigma^* : |x| = 2r(x) \}.
\]

Show that \( L \) is a CFL.
Additional Problems — if you already solved your group problem try these

10. Can you construct an aperiodic infinite word in which there are powers of arbitrarily large exponent beginning at every position? Hint: construct it iteratively.

12. When is the concatenation of two antipalindromes a palindrome? Give necessary and sufficient conditions. Possible strategy: do some experiments.

20. Can you find a solution in nonempty words to the equation \( x^2 y^2 z^2 = w^2 \) where no pair of words chosen from \( x, y, z, w \) commutes?

30. If \( \text{shuff}(L, \{0\}) \) is regular, need \( L \) be regular?

54. Give an example of a language \( L \) (evidently it must be non-regular) where \( RL \) has infinitely many equivalence classes, and each one has infinitely many members.