University of Waterloo
CS 462 — Formal Languages and Parsing
Winter 2021
Problem Set 7

Distributed Monday, March 8 2021.
Due Thursday, March 18 2021 by 5 PM. Note different day! Hand in to LEARN.

We will discuss the subsequence order, and sub and sup, and closure properties of CFL’s in the lecture of Wednesday March 10, so either read ahead in the course notes, or wait until then.

1. [10 marks] Consider the language

\[ \text{PRIMES}_6 = \{2, 3, 5, 11, 15, 21, 25, \ldots \}, \]

the prime numbers expressed in base 6. (Note: 1 is not a prime, and no leading zeros are allowed!) Find the set of minimal elements for the subsequence order for this language. Hint: for the kind of case analysis you might try, see Example 3.13.6 and my article, “Minimal Primes”, available at

https://cs.uwaterloo.ca/~shallit/Papers/minimal5.pdf

which deals with the case PRIMES_{10}. (But the analysis in that paper is much more detailed than what is necessary for PRIMES_6.)

2. [10 marks]

(a) Express the quotient of languages \( L/R \) in terms of the operations of morphism, inverse morphism, and possibly other operations such as concatenation and intersection.

(b) Show that if \( L \) is a CFL and \( R \) is a regular language, then the quotient \( L/R \) is a CFL.

3. [10 marks] Let \( L \subseteq \Sigma^* \). Do there exist algorithms to compute DFAs \( D \) and \( D' \) such that \( L(D) = \text{sup}(L) \) and \( L(D') = \text{sub}(L) \) if

(a) \( L \) is recursively enumerable (that is, Turing-recognizable) and given to you as \( L(T) \) for a one-tape deterministic TM \( T \) provided as input?

(b) \( L \) is regular, and given to you as \( L(M) \) for a DFA \( M \) provided as input?