

Assignment 2

Due October 26 at noon

For all problems you are expected to justify your answers, by showing your work or stating arguments, as is appropriate.

The solutions you hand in must be entirely your own work. Do not look up either full or partial solutions in the literature or on the Internet.

Please read course policies and the course Web page for more information.

- [20 marks]* In this question you will show that problem A is linearly reducible to problem B and that problem B is linearly reducible to problem A, where A is the problem of squaring an upper triangular matrix whose diagonal consists only of 1's, and B is the problem of multiplying two square matrices. For each reduction, specify which additional assumptions, if any, are needed to complete the proof.
- [20 marks]* Consider a sorting problem in which you are given a partially sorted sequence of n elements as input. That is, the first m elements are all smaller than elements $m + 1$ to n , the first $2m$ elements are all smaller than elements $2m + 1$ to n , and in general, the first im elements are all smaller than elements $im + 1$ to n . You may assume that n is a multiple of m .
 - [10 marks]* Prove an upper bound of $O(n \log m)$.
 - [10 marks]* Prove a lower bound of $\Omega(n \log m)$.
- [20 marks]* Consider the problem of determining whether or not there exists a path of length at least two in an undirected graph (recall that the length of a path is the number of edges in the path). Prove a lower bound of $\Omega(n^2)$ on any algorithm which can only make queries of the form "Is (x, y) an edge?"
- [20 marks]* Determine an adversary lower bound of $\binom{n}{\lfloor n/2 \rfloor}$ for the following problem. Given n objects with real weights and a weight W , determine whether or not there is a subset of the objects of total weight exactly W , where each query consists of specifying a subset S of the objects, and the answer "=", "<", or ">" is returned depending whether or not the sum of the weights of the objects in S is equal to, less than, or greater than W .
- [20 marks]* Consider an array A of size n storing values in the set $\{1, \dots, 10\}$, and suppose that you can access A by asking for the value of a particular location in the array. Determine the best upper and lower bounds on the number of queries needed to determine whether or not there are consecutive locations containing 5's. For full credit you should be able to obtain matching bounds. Hint: you may wish to consider contiguous subarrays of A .