CS 466/666 Fall 2004
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Assignment 4
Due November 30 at noon

For all problems you are expected to justify your answers, by showing your work or stating arguments, as is appropriate.

The solutions you hand in must be entirely your own work. Do not look up either full or partial solutions in the literature or on the Internet.

Please read course policies and the course Web page for more information.

1. *[60 marks]* For each of the problems given below, prove that the problem is fixed-parameter tractable. You do not need to give details of the running time analysis other than to show that part is a function of the parameter only and that the rest is polynomial in the input size.

   (a) In the problem below, an *exchange* operation at position i results in the symbols at positions i and $i+1$ being interchanged.

   Instance: A finite alphabet Σ, a string $x \in \Sigma^*$, and a nonnegative integer k.

   Parameter: k.

   Question: Is there a sequence of at most k exchange operations that will change x into a string y in which, for each symbol $a \in \Sigma$, all occurrences of a appear consecutively in y?

   (b) **Instance:** An undirected graph $G = (V, E)$, a positive integer k.

   Parameter: k.

   Question: Are there k vertex-disjoint 3-cycles in the graph?

   (c) **Instance:** An undirected graph $G = (V, E)$ and a positive integer k.

   Parameter: The treewidth w of G.

   Question: Is there a subset $V' \subseteq V$ of size at least k such that $G' = (V', E)$ is acyclic?

2. *[20 marks]* Given a decision problem P and two Monte Carlo algorithms A and B for P, we wish to construct a Las Vegas algorithm C. You should assume that algorithm A returns a correct answer with probability $2/3$, and is always correct when the answer returned is `true` and B returns a correct answer with probability $3/4$, and is always correct when the answer returned is `false`. Moreover, the worst-case running times of A and B are $3n$ and $5n$ respectively, where n is the size of the input. Describe the algorithm C (in pseudocode or succinct English) and briefly determine an upper bound on its expected running time (as precise as possible). In your analysis you may find it convenient to consider inputs with correct answer `true` separately from those with correct answer `false`.

3. *[20 marks]* Use a potential argument to prove that flush-when-full (FWF) is k-competitive, where k is the size of the cache. In this paging algorithm, whenever a page fault occurs when the cache is full, the entire cache is emptied. Hint: let C be the set of pages in OPT's cache, let D be the set of pages in FWF's cache, and let $\Phi = |D|(k-1) + |C - D|k$. The "actual" cost of an algorithm is 1 if there is a page fault and 0 otherwise.