CS 466/666
Assignment 1
Fall 2005
Due date: Friday October 7th, 6:00pm

Part 1 Models of computation

1. *Bargain Computers Corporation* released a RAM machine with words that are one bit long. All the basic machine instruction operations (AND, OR, CMP, etc) take as operands one bit values and compute the respective operation. Given a binary number *m* bits long as input, describe an efficient algorithm in such a computer that returns 0 if the input is all zeroes, and 1 otherwise. Give the time analysis.
2. *Fancy Computers Corporation* released a RAM machine with words that are *m* bits long. Given a binary number *m* bits long as input, describe an efficient algorithm in such a computer that returns 0 if the input is all zeroes, and 1 otherwise. Give the time analysis.
3. Using a computer as in question 2 above, and given a binary number *m* bits long as input, describe an efficient algorithm to compute the number of 1 bits in the input under each of the following assumptions:
    a. The total amount of memory available is $O(2^m)$ bits in size
    b. The total amount of memory available is O(m) bits in size

Part 2 Amortized analysis

4. In class we proved that $\text{rank}(n) \leq n - 1$ for UNION-FIND with path compression and union-by-rank. Prove that in fact $\text{rank}(n) \leq \log_2(n)$.
5. Show that if all UNION operations precede all FIND operations, then a sequence of k UNION-FIND operations using path compression and weighted union takes O(k) time.