CS 466/666
Assignment 4
Due: Noon Thursday November 13, 2008

1) [5 marks] The **polar angle** of a point p_1 with respect to an origin point p_0 is the angle of the vector $p_1 - p_0$ in the usual polar coordinate system. For example, the polar angle of (3,5) with respect to (2,4) is the angle of the vector (1,1), which is 45 degrees. The polar angle of (3,3) with respect to (2,4) is the angle of the vector (1,-1), which is 315 degrees. Write pseudocode to sort a sequence $(p_1, p_2, \ldots, p_n)$ of n points according to their polar angles with respect to a given origin point p_0. Your procedure should take O(n log n) time and use cross products to compare angles.

2) [15 marks] Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the line segments of S. We wish to determine all line segments of S that p can see, that is, all line segments of S that contain some point q so that the open segment pq does not intersect any line segment of S. Give an O(n log n) time algorithm for this problem that uses a rotating half-line with its endpoint at p.

3) [15 marks] Let $P=(p_0, p_1, \ldots, p_{n-1}, p_n = p_0)$ be a non-convex polygon which is simple (its edges $p_i p_{i+1}$ do not intersect). Describe a linear-time algorithm that computes the convex hull of P.

4) [5 marks] Consider the language L of Boolean expressions that do not contain any "not" operations. One can define this language as the set of strings over alphabet $\Sigma = \{$a,0,1, "and", "or"$\}$ as follows:

$$Var \rightarrow a\{0,1\}^*$$

$$Exp \rightarrow Var \mid (Exp) \mid Exp \text{ "and" } Exp \mid Exp \text{ "or" } Exp$$

    Show L is in P.

5) [10 marks] Show that the problem of determining the satisfiability of Boolean formulas that contain no "not" operations is in P.

6) [10 marks] Show that the problem of determining the satisfiability of Boolean formulas in DNF (Disjuncive Normal Form ... the "or" of clauses of "ands" even if they containing negations of literals) is in P.