## CS 466/666 Assignment 5

Due: Noon Thursday November 27, 2008

- 1) [10 marks] Give a linear time algorithm that finds an optimal vertex cover for the special case that the graph is a tree.
- 2) [15 marks] Consider the following greedy algorithm for finding a vertex cover on a graph: Sort the vertices of the graph in non-increasing order according to the degrees. Iteratively include the next vertex on the list to the vertex cover and verify if the set is indeed a vertex cover. If so, declare the set as a vertex cover, otherwise continue with the next vertex on the list (note that no edge is deleted in the process). Show that the algorithm does not necessarily approximate the optimal vertex cover by **ANY** constant factor.
- 3) [10 marks] Suppose that the vertices of an instance of the traveling-salesman problem are points in the plane and the cost of an edge is the Euclidean distance between its endpoints. It was claimed in class that an optimal tour never crosses itself. Prove this.
- 4) [15 marks] Given an  $m \times n$  integer matrix A and an integer m-vector b, the "Boolean coding" problem asks whether there is an integer n-vector x with elements in the set  $\{0,1\}$  such that  $Ax \le b$ . Prove that "Boolean coding" problem is NP-complete (Hint: reduce from 3-SAT).