CS 466/666
Assignment 4
(Due: Noon Friday November 19, 2010)
Justify all answers

1. [10 marks] Consider the language L of Boolean expressions that contain no "not" operations. One can define this language as the set of strings over alphabet Σ = {a,0,1, "and", "or"} as the BNF (or context free grammar) below. (The superscript "+" denotes an arbitrary finite non empty string of the symbols in curly braces, e.g. 100101.):

   i. Var $\rightarrow$ a$\{0,1\}^+$

   ii. Exp $\rightarrow$ Var | (Exp) | Exp "and" Exp | Exp "or" Exp

   a) Show L is in P. (Note this part deals with the picky issue of syntactic correctness, that we sometimes, though formally should not, ignore.)

   b) Show that the problem of determining the satisfiability of Boolean formulas that contain no "not" operations is in P. (Don't forget to use part a).)

2. [5 marks] Show that the problem of determining the satisfiability of Boolean formulas in DNF (Disjunctive Normal Form ... the "or" of clauses of "ands" containing literals is in P.

   (A literal is a variable or its negation. You may assume each formula is syntactically correct. Cook originally worked with DNF, though his interest was in "tautologies", statements true for all assignments of the variables, rather than "satisfiability".)

3. [5 marks] Consider the problem of determining the satisfiability of Boolean formulas in CNF (conjunctive normal form) in which at most lg n (of n or so) clauses contain 3 literals, the remaining clauses contain only 2 literals. Show this problem is in P. You may assume each formula is syntactically correct.