

Assignment 6.

Post date: Nov. 11, 2014.

Due date: Nov. 19, 2014 (Wednesday) by noon. Hand in printed paper at assignment box at 4th floor of MC across from the tutorial centre.

Marking TA: Chuan Guo

Mandatory requirement for assignment being accepted:

The answers need to be typed up with a computer and printed on letter size paper. Only minor corrections can be made with handwriting. The first page of the submission must be a cover page that contains your name, student id, and course number (CS466 or CS666) and assignment number.

Question 1. (15 marks)

In a weighted graph, a *simple path* is a series of connected edges that can visit each vertex *at most* once. It is okay if the simple path does not visit all vertices. The length of a path is equal to the sum of the weights of all edges on the path. The Longest Path Problem finds a simple path in the graph with the maximum length.

- (5 marks) Write the decision version of the Longest Path Problem.
- (10 marks) Prove that the decision version is NP-complete. The reduction should be from Hamiltonian Path problem, which is already known to be NP-complete.

Question 2. (30 marks)

Let $G = \langle V, E \rangle$ be an undirected, weighted complete graph. Here V is the vertex set and E is the edge set. Each edge $e = (u, v)$ has a weight $d(u, v) > 0$. The edge weight satisfies Triangular Inequality. That is $d(u, v) + d(v, w) \geq d(u, w)$.

Let $V' \subset V$ be a subset of vertices. Let $G' = \langle V', E' \rangle$ be the complete subgraph that contains all vertices in V' , and $E' = \{(u, v) \in E \mid u \in V' \text{ and } v \in V'\}$.

A Steiner tree is a connected acyclic subgraph of G , such that all vertices V' are connected in the tree. The weight of a tree T , denoted by $w(T)$ is the total weight of all edges on the tree. The Minimum Steiner Tree problem asks for the minimum weighted Steiner tree.

Prove the following:

- (10 marks) Suppose T is a minimum Steiner tree, there is a path P such that all vertices in V' are on the path, and the weight of the path is at most twice of the weight of T . (Hints: The path is not necessarily a simple path. Use a depth-first traversal on T .)
- (10 marks) There is a spanning tree T' of G' , such that the weight of T' is at most the weight of P . (Hint: Replace some segments of P with edges in G' . Break circles if needed.)
- (10 marks) Design a polynomial time ratio-2 approximation algorithm for Minimum Steiner tree problem. Use conclusions from a) and b) to prove the ratio.