CS466 — Fall 1999

Problem Set 4

These problems are due Thursday, Dec. 2, in class. Do your own work.

1. [5 marks]
   Consider the following algorithm to find an approximation to the maximal cut in a graph.

   Input: $G = (V, E)$

   $C \leftarrow \emptyset$

   Repeat

       If there is a vertex x not in C such that $C \cup \{x\}$ is a larger cut than C then
           add x to C

       Else if there is a vertex x in C such that $C - \{x\}$ is a larger cut than C then
           remove x from C

       Else return C

   Show that the cut C returned by the algorithm has size at least one-half the size of the maximal cut in G.

2. [10 marks]
   CLR Problem 37-1, pp. 983–4, parts b through f (omit part a, on NP-hardness).

3. [5 marks]
   CLR Problem 37-2, p. 984.

4. Consider the following problem on a PRAM. The first n processors each receive one bit of input and all memory locations have initial value 0. The PRAM (i.e., processor 1) should accept if any of the input bits has value 1 and reject otherwise.

   Assume that each processing cycle consists of local computation, followed by an optional write, followed by an optional read.

   (a) [1 mark]
   Show that the problem can be solved in one compute-write-read cycle using concurrent reads and/or writes and in $\log_2 n$ cycles using exclusive reads and writes.

   (b) [9 marks]
   Consider the following argument that computing OR using exclusive reads and writes requires $\log_2 n$ cycles.

   > In order for processor 1 to accept or reject correctly, it must "know about" the input of each of n processors. Let $K(i,t)$ be the number of processors that processor i knows about after t cycles of the algorithm. We show by induction that $K(i,t) \leq 2^t$. Obviously, $K(i,0) = 1$ for all i. At step t, processor i reads from some memory cell j, and thereby learns the information written there at some previous time t' by some processor i'. Hence $K(i,t) \leq K(i,t-1) + K(i',t') \leq 2^{t-1} + 2^{t'} \leq 2^t$. The algorithm can halt correctly after t steps only if $K(1,t) = n$. Hence $t \geq \log_2 n$.

   i. What error does the explanation make?
   ii. Give an algorithm that accepts five-input OR in two cycles using exclusive writes. Conclude that there is an algorithm for n-input OR that takes less than $0.862 \log_2 n$ cycles. (Alternatively, give any algorithm that uses less than $c \log_2 n$ cycles, for come $c < 1$ and all sufficiently large n.)
   iii. Show that OR requires $\Omega(\log n)$ cycles using exclusive writes.
   iv. [optional] For extra credit, find the exact constant for matching upper and lower bounds.