CS 466/666, Spring 2006
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Assignment 5 (due July 26 Wednesday(!) noon)

(There are only 3 questions in this assignment, all coincidentally involving the number 3...)

1. [*13 marks*] Consider the following job scheduling problem: we have $n/3$ processors available, each capable of handling 3 jobs; we would like to assign jobs to processors so as to minimize the overall (maximum) completion time. More precisely, given a sequence of n positive numbers $s_1, \ldots, s_n$ (where n is divisible by 3), we would like to partition the sequence into subsets $S_1, \ldots, S_{n/3}$, each containing 3 numbers, so as to minimize the quantity $\max_{j=1,\ldots,n/3} \sum_{s \in S_j} s$.

   (a) [*3 marks*] Let k be a fixed constant and let A be a fixed set of k elements. Show that in the special case where all s_i's come from the set A (duplicates are allowed), the problem can be solved exactly in polynomial time.

   (b) [*10 marks*] Now design and analyze a polynomial-time approximation scheme (PTAS) for the general problem. [Hint: use a rounding technique (but it should be simpler than the bin-packing PTAS from class).]

2. [*12 marks*] We are given a collection $\mathcal{A}$ of subsets of $\{1, \ldots, n\}$, where each subset has size 3. For each $i \in \{1, \ldots, n\}$, we are given a cost $c_i > 0$. We would like to find a subset $S \subseteq \{1, \ldots, n\}$, minimizing total cost $c(S) := \sum_{i \in S} c_i$, subject to the constraint that S intersects every subset of $\mathcal{A}$, i.e., $S \cap A \neq \emptyset$ for all $A \in \mathcal{A}$.

   For example: if $\mathcal{A} = \{\{1,3,4\}, \{2,3,5\}, \{3,4,5\}, \{1,2,5\}\}$ and $c_1 = c_2 = \cdots = c_5 = 1$, then an optimal solution is $S = \{3, 5\}$ with cost 2.

   Design and analyze a deterministic polynomial-time approximation algorithm of factor at most 3. [Hint: First formulate an integer program with n variables, where $x_i = 1$ means $i \in S$. Then solve its linear programming relaxation. Don't use randomized rounding but define a subset S that contains all indices i with $x_i \geq 1/3$. Argue that this subset is a feasible solution...]

3. [*10 marks*] Consider a variant of the lost cow problem from class: instead of a cow in search of the fence, you are in search of the cow. Assume that the dimension is 1 (you and the cow lie on the real line), the cow is stationary, and you are near-sighted. Let d denote the distance between your initial location and the cow (d is a positive integer). You don't know d, nor the direction (left or right) where the cow is. However, you know that there is a bicycle located 32 units to the right of your initial location. The bicycle allows you to travel 3 times as fast; namely, your walking speed is 1 unit per minute but your bicycling speed is 1/3 unit per minute. Your objective is to minimize travel time (instead of distance).

   Show that there is a deterministic algorithm with competitive ratio at most 9 for this problem. [Hint: although probably the same algorithm from class works, the analysis requires change. In particular, the cost of the optimal offline solution is not d, but a different expression (at least two cases arise)...]