CS 466/666 Spring 2013
Assignment 10
Due Noon, July 29, 2013

You are on your honour to present your own work and acknowledge your sources.

1. [14 marks] This question deals with an algorithm to find the second largest of n values.

   Take a random sample, R, of r of the values; r = n/lg n (Assume r is a power of 2).
   By repeatedly pairing the elements that have "won" every comparison, find the maximum of the sample (in r-1 comparisons). Initialize m_1 as the sample maximum and m_2 as the element that was the maximum of half the sample but "lost" to the maximum in the last comparison.
   Scan through the remaining elements (i.e. those not in R), comparing each to m_2 and if it is larger than m_2 compare it with m_1. Update m_1 to be the maximum seen so far and m_2 to be the second largest (perhaps excluding some of the values from the R).
   When finished the scan, m_1 is the maximum of the set; under <u>some condition</u>, you may know m_2 is the second largest value in the set. If you cannot guarantee m_2 is the second largest, find the maximum of m_2 and the random sample elements that "lost" directly to the original m_1.

a. [2 marks] What is the maximum number of comparisons this method could use? (Briefly justify the claim.)
b. [4 marks] What condition, detectable in this algorithm, would guarantee that m_2 is the second largest after finishing the scan of all elements? What is the probability that this condition will hold? (Briefly justify this claim)
c. [2 marks] If the condition above is not satisfied, how many more comparisons are required?
d. [4 marks] What is the expected number of times m_2 is replaced (i.e. the number of times a second comparison is done) in scanning the n-n/lg n non sample elements? (Give the expected number of times including the constant factor in the lead term, and justify the claim)
e. [4 marks] Given the probability of the condition being satisfied and the expected number of comparisons used in that case, and the probability of it not being satisfied and the number of comparisons used in the latter case, what is the expected number of comparisons used by the method. (Justify this claim.)