

CS 466/666 Spring 2013
Assignment 9
Due Noon, July 22, 2013

You are on your honour to present your own work and acknowledge your sources.

1. [14 marks] Consider Mergesort when n is not (necessarily) a power of 2. The method works by (recursively) sorting a subarray of size $\lceil n/2 \rceil$ and one of size $\lfloor n/2 \rfloor$ and then merging them in $n-1$ comparisons. A segment of length 1 requires 0 comparisons.
 - a. [2 marks] Give a recurrence relation that describes the number of comparisons used, in the worst case, by this method.
 - b. [4 marks] Prove that $n-1$ comparisons are necessary (i.e. you cannot do it in fewer), in the *worst case* for this merge step
 - c. [4 marks] Prove that Mergesort, as described above, takes $n \lceil \lg n \rceil - 2^{\lceil \lg n \rceil} + 1$ comparisons in the worst case.
 - d. [4 marks] The *expected* number of comparisons for this method (over all possible permutations of the input) is a little ($\Theta(n)$) better. Prove it. (You do not have to deal with the exact constant in this $\Theta(n)$ term.)