CS 466/666 Spring 2014
Assignment 5
Due Noon, June 16, 2014

You are on your honour to present your own work and acknowledge your sources.

1. [14 marks] Consider Mergesort when n is not (necessarily) a power of 2. The method works by (recursively) sorting a subarray of size $\lceil n/2 \rceil$ and one of size $\lfloor n/2 \rfloor$ and then merging them in n-1 comparisons. A segment of length 1 requires 0 comparisons.
   a. [2 marks] Give a recurrence relation that describes the number of comparisons used, in the worst case, by this method.
   b. [4 marks] Prove that n-1 comparisons are necessary (i.e. you cannot do it in fewer), in the *worst case* for the merge step.
   c. [4 marks] Prove that Mergesort, as described above, takes $n\lceil \lg n \rceil - 2^{\lceil \lg n \rceil} + 1$ comparisons in the worst case.
   d. [4 marks] The *expected* number of comparisons for this method (over all possible permutations of the input) is a little ($\Theta(n)$) better. Prove it. (You do not have to deal with the exact constant in this $\Theta(n)$ term.)