1. (a)

We are asked to compute

$$P(x=1 \mid y=1)$$

We have

$$P(y=0 \mid x=0) = 0.90 \quad (1)$$

$$\Rightarrow P(y=1 \mid x=0) = 0.10 \quad (2)$$

$$P(y=1 \mid x=1) = 0.99 \quad (3)$$

$$\Rightarrow P(y=0 \mid x=1) = 0.01 \quad (4)$$

(a) $P(x=1) = 10^{-3} = 0.001$ (5)

$\Rightarrow P(x=0) = 1 - P(x=1)$ (6)

We are asked to compute

$$P(x=1 \mid y=1)$$

Bayes Rule: $P(A|B) = \dfrac{P(B|A)P(A)}{P(B)}$ √(3) √(5)

$$\Rightarrow P(x=1|y=1) = \frac{P(y=1|x=1)P(x=1)}{P(y=1)}$$

we need

$P(y=1) = ?$

?

Law of Total Probability

$$P(A) = \sum_i P(A \mid B_i) P(B_i)$$

$x \in \{0, 1\}$

$$P(y=1) = P(y=1 \mid x=0)\,(1 - P(x=1)) + P(y=1 \mid x=1)\,P(x=1)$$

(2) (3)

$$P(y=1) = (0.10)(1 - P(x=1)) + (0.99)\,P(x=1) \quad (7)$$

(3)

$$\Rightarrow P(x=1|y=1) = \frac{P(y=1|x=1)P(x=1)}{P(y=1)} \quad (7)$$

$$= \frac{(0.99)\ P(x=1)}{(0.10)(1-P(x=1)) + (0.99)P(x=1)}$$

(a) $P(x=1) = 10^{-3}$

$$P(x=1|y=1) = \frac{(0.99)(10^{-3})}{(0.10)(1-10^{-3}) + (0.99)(10^{-3})}$$

$$= 0.0098127$$

$$\approx 9.81 \times 10^{-3}$$

Think about what this means

(b) $P(x=1) = 10^{-2}$

$$P(x=1 \mid y=1) = \frac{(0.99)(10^{-2})}{(0.10)(1-10^{-2}) + (0.99)(10^{-2})}$$

$$= 0.090\,91$$

$$\approx 9.09 \times 10^{-2}$$

(c) $P(x=1) = 10^{-1}$

$$P(x=1 \mid y=1) = \frac{(0.99)(10^{-1})}{(0.10)(1-10^{-1}) + (0.99)(10^{-1})}$$

$$= 0.524$$

$$\approx 5.24 \times 10^{-1}$$