CS 762 Winter 2011 due Wed. Feb. 9

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ASSIGNMENT 1

You will get more out of the problems if you try to do them yourself without looking them up anywhere. If you use any resources (books, papers, web sites, etc.) be sure to acknowledge them. And be sure to write the solution in your own words—the best way to be sure of this is to put any resources you may have used out of sight when you write your solution.

1. This problem is about an alternative way to triangulate a planar graph. Recall that a graph is *outerplanar* if it is planar with all vertices on the outer face.

   (a) Prove that for any 2-connected outerplanar graph G and any edge (u, v) on the outer face of G, there is a vertex $w \neq u, v$ of degree 2.

   (b) Using this result, prove that any planar graph can be triangulated by taking every face of $k \geq 4$ vertices and adding $k-3$ edges all incident to one vertex of the face.

   (c) Give a linear time algorithm to triangulate as in (b). If you cannot get linear time, then give the best algorithm you can. Be sure to analyze your algorithm.

2. This problem is about a variant of Menger's theorem. Let G be an undirected triangulated planar graph, with distinct vertices $s_1, \ldots, s_k, t_k, \ldots, t_1$ appearing in that order around the outer face. Prove that either there are k vertex disjoint paths s_i to t_i or there is a path of length less than k whose removal separates the s_i's from the t_i's. [As I mentioned in class, we will use this in the proof of the planar separator theorem. In fact, we will just need the case where $s_k = t_1$ and $t_k = s_1$ and the path (of necessity) joins these two.]

3. We defined a *realizer* of a triangulated planar graph to be an assignment of directions and colours (red, green, blue) to the internal edges such that the following two properties hold:

   (a) for every internal vertex v, there is one outgoing edge of each colour and ≥ 0 incoming edges of each colour, and the pattern, in clockwise order is: outgoing red, incoming green, outgoing blue, incoming red, outgoing green, incoming blue.

   (b) the vertices of the outer face are coloured red, blue, and green in clockwise order. The internal edges incident to the vertex of colour X are directed towards the vertex and all have colour X.

   Prove that in fact condition (a) implies condition (b). Hint: start by counting internal edges, and prove that the edges have the required directions; then deal with colours.